## Underdominance Predictions and Genetically Transforming a Population

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### **Genetic Pest Management (GPM)**

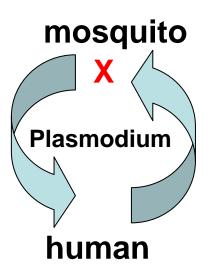
Many diseases vectored by insects are taking a huge humanitarian toll

Malaria kills 1 million people a year, antimalarial drug resistance is evolving and spreading, and no effective vaccine has been developed.

Dengue fever is rapidly spreading, no effective vaccine has been developed.

West Nile virus is spreading across N. America, no vaccine for humans exists.

Gould 2006

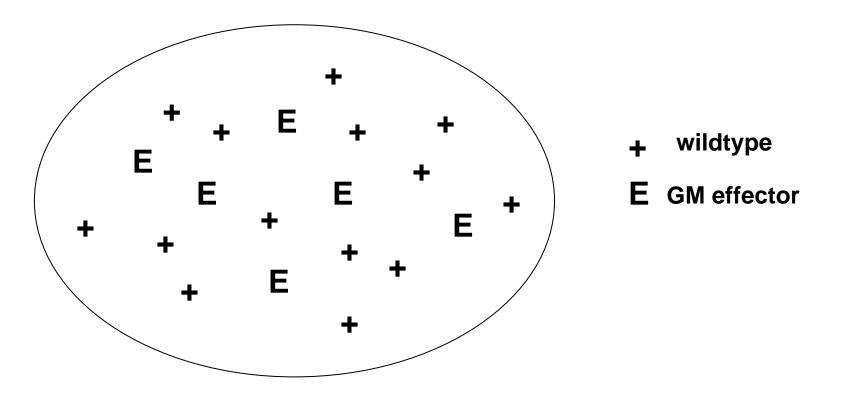


In theory, an insect that vectors a harmful disease can be genetically modified to be resistant to infection and no longer transmit the disease.

There has been success in developing refractory constructs in *Anopheles* against *Plasmodium falciparum* (Corby-Harris *et al*. 2010), and in *Aedes* against *P. gallinaceum* (Jasinskiene *et al*. 2007).

We can refer to this genetic modification as an "effector."

Effectors can be released into a wild population:



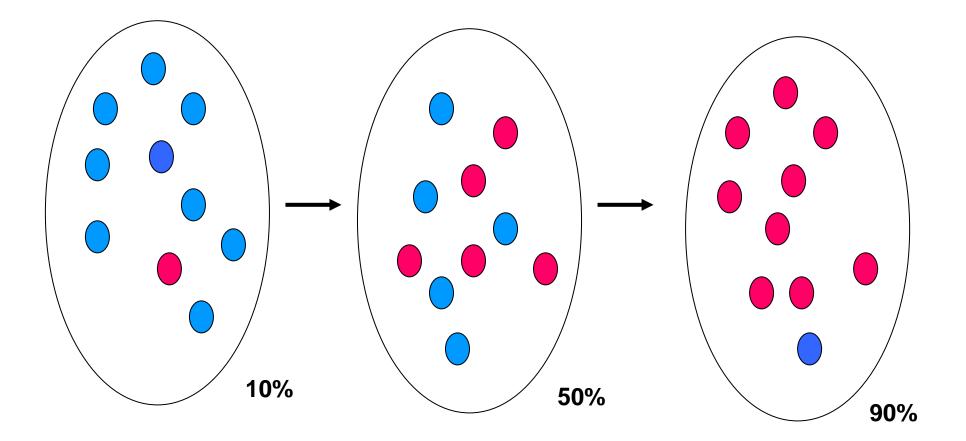
But, if there is no fitness advantage, and especially likely, if there is a fitness cost, they are unlikely to reach fixation and may be quickly lost from the wild.

It may be next to impossible to engineer a construct with higher fitness than wildtype (in the adaptive sense).

However, there are types of selfish genetic elements that can increase in frequency without an adaptive fitness advantage (*e.g.* meiotic drive, TEs, Wolbachia, Medea, ...?).

These may be utilized as "drive" mechanisms linked to effector constructs to push effectors to high frequency or fixation in the wild.

### The Wright--Fisher "bean bag" model.



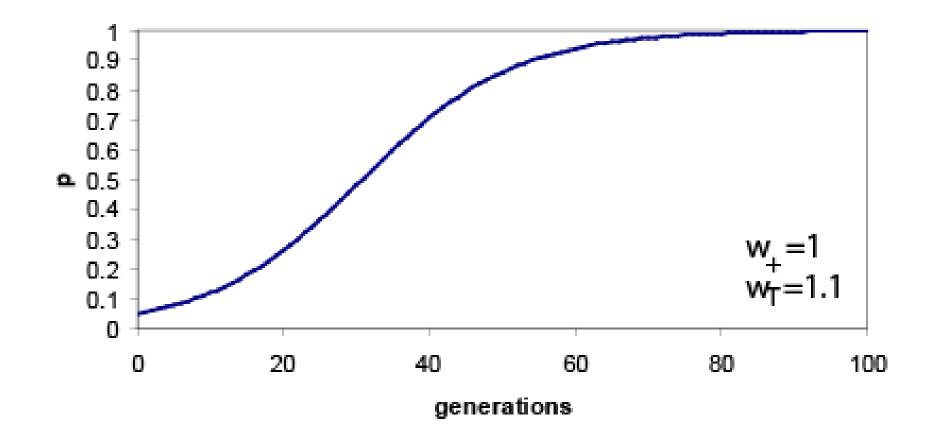
Natural selection changes allele frequencies in a population over time.

Say there is a mutant allele, T, at a frequency of p with a fitness of  $w_T$ . The predicted frequency in the next generation is simply its frequency multiplied by fitness and normalized by the average fitness in the population.

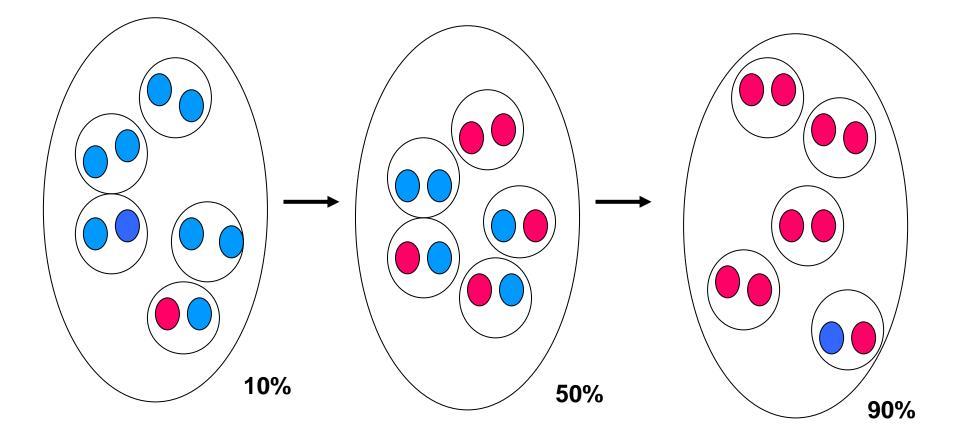
$$p' = \frac{w_T p}{\overline{w}}$$

If an allele has a higher than average fitness it will increase in the population.

With a simple fitness advantage, one allele replaces the other over the following generations.



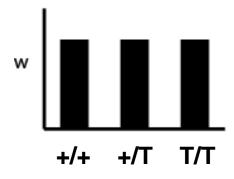
### Of course things are not so simple (diploids)



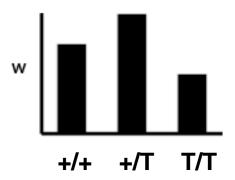
In diploids alleles are paired into individuals, rare alleles tend to be heterozygous.

# Two alleles give three genotypes and four different fitness configurations.

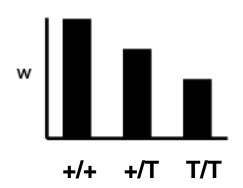
### 1) Neutrality



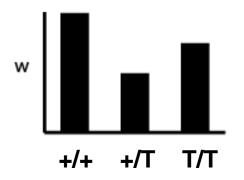
3) Overdominance

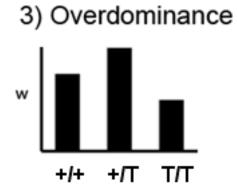


2) Directional

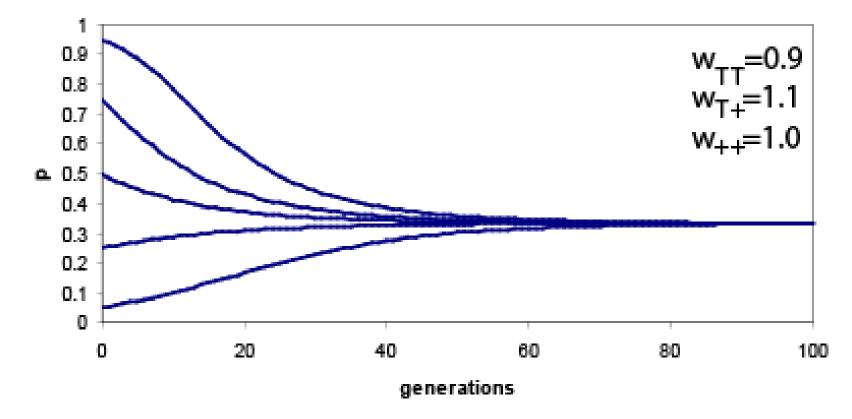


4) Underdominance

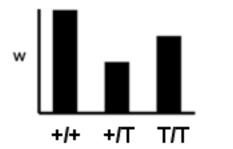




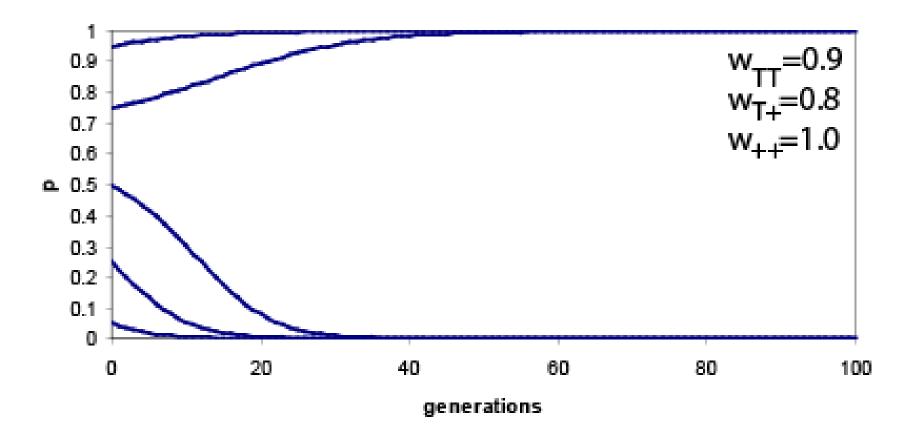
A heterozygote advantage leads to a stable equilibrium.



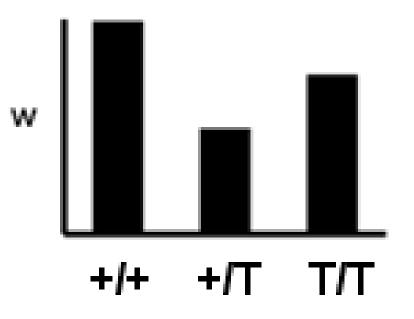
#### 4) Underdominance



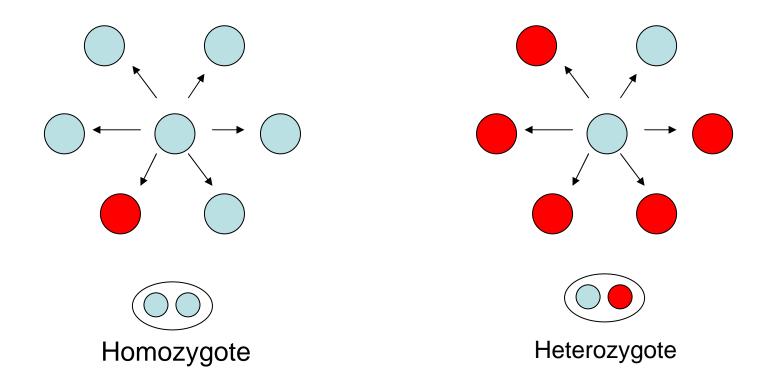
A heterozygote disadvantage leads to an unstable equilibrium.



Because of this bistable nature, underdominance can be an alternative to the types of "selfish drive" systems mentioned before.

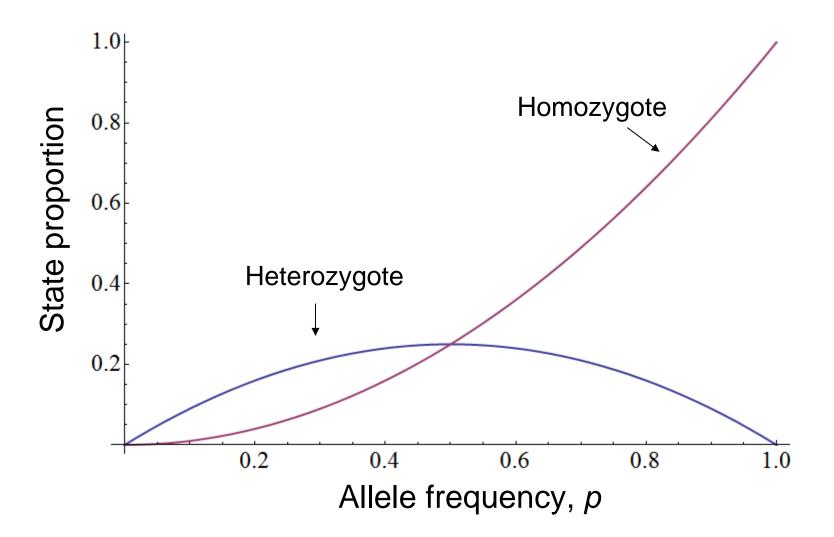


Possible allele pairings in a population

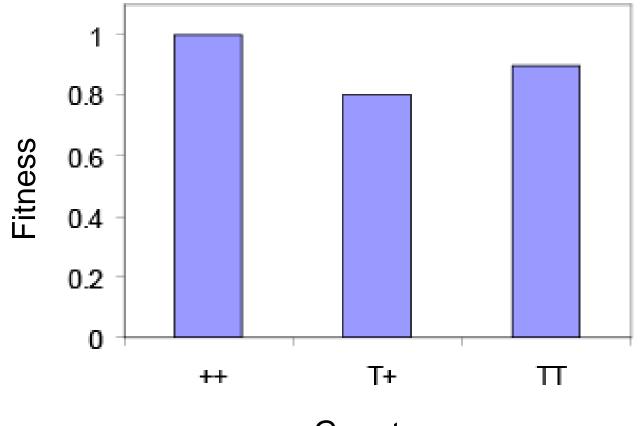


Common alleles are present most often as homozygotes, rare alleles are present most often as heterozygotes.

A rare allele spends more time as a heterozygote than a homozygote

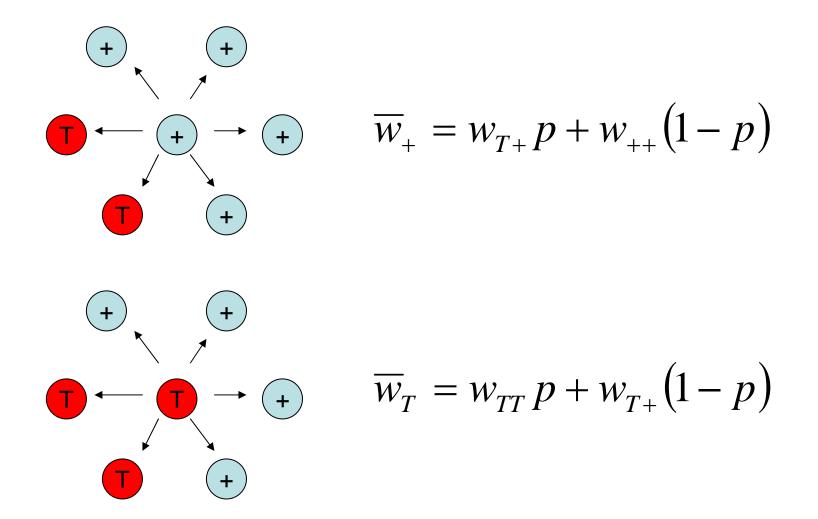


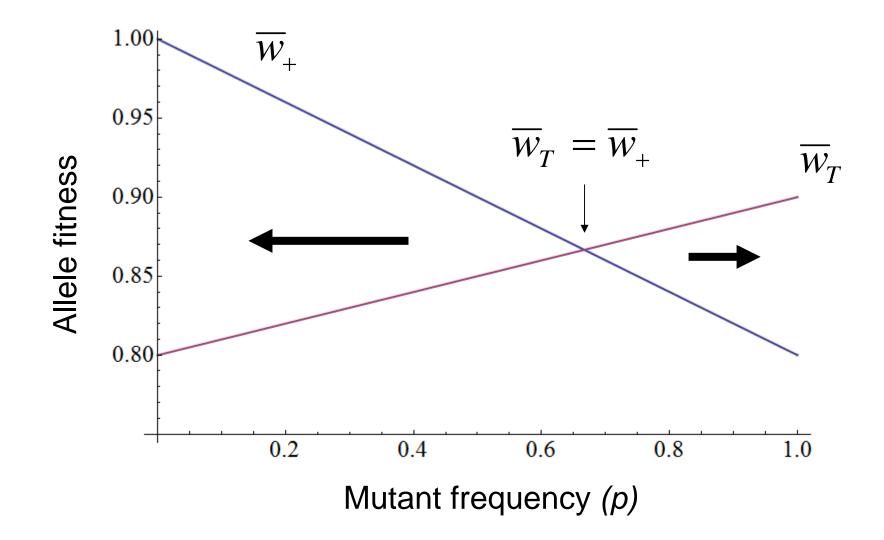
A heterozygote disadvantage corresponds to a rare allele disadvantage



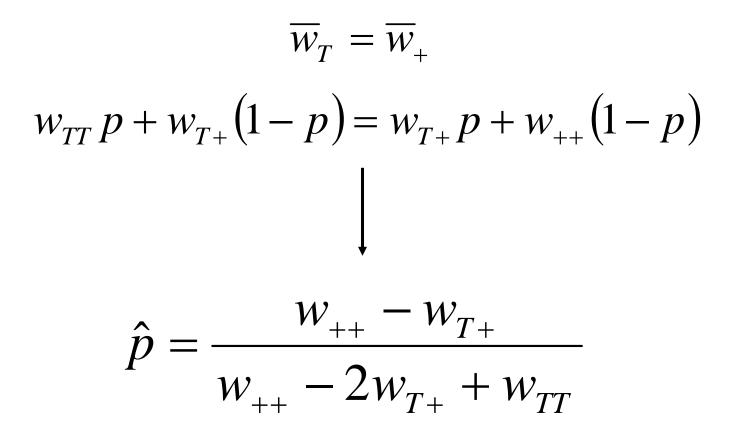
Genotype

We can estimate the average fitness of each allele

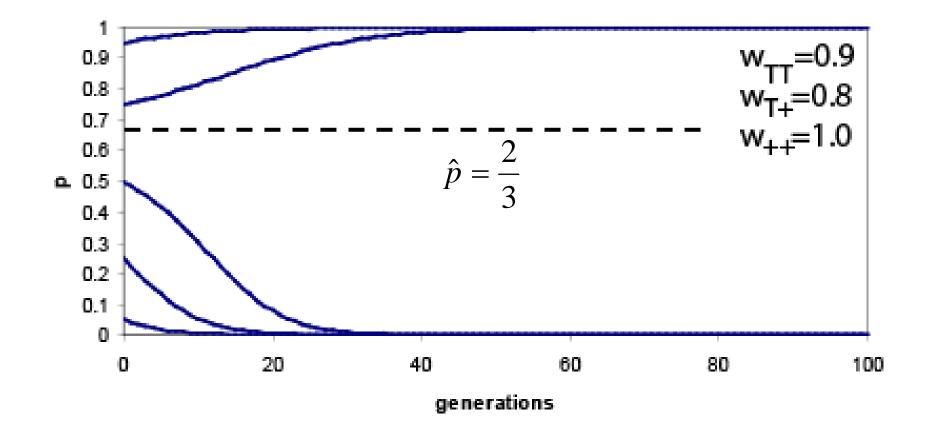




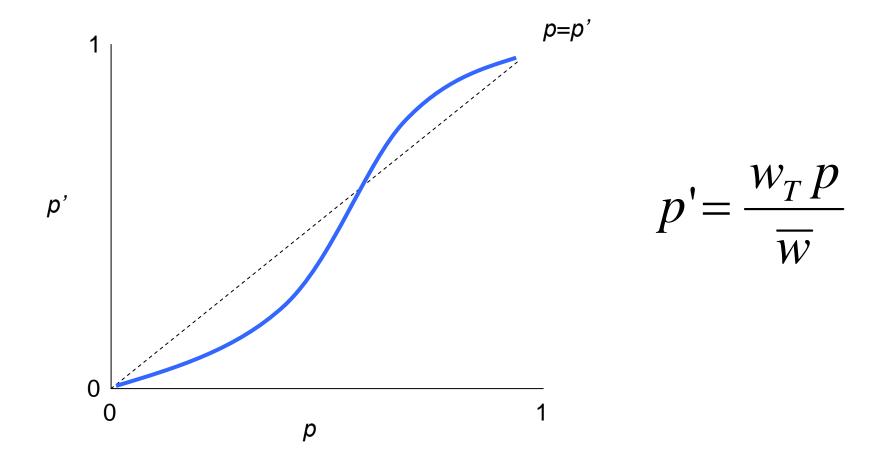
### **Equilibrium value**



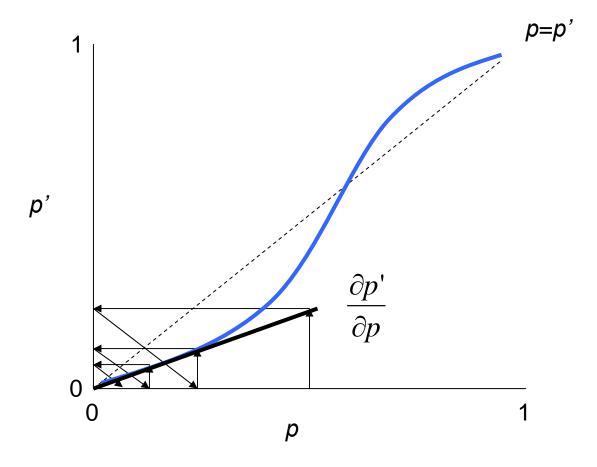
If starting at a frequency above this value, an allele <u>less fit than wildtype</u> can stably fix in a population.



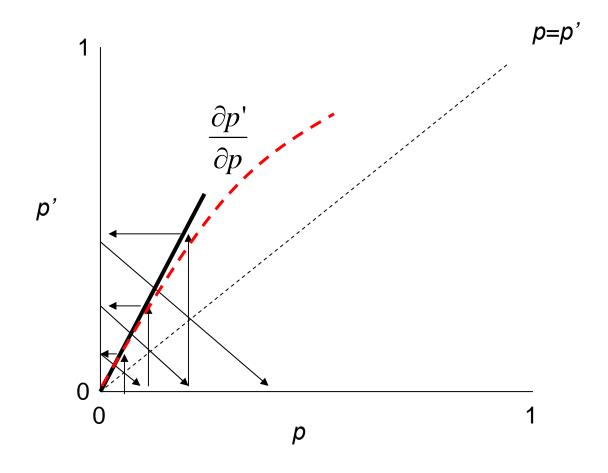
### A different way to visualize underdominance



### Stable equilibrium at *p*=0



Unstable equilibrium at *p*=0



To describe the stability near the point p=0Find  $\frac{\partial p'}{\partial p}$ Substitute in *p*=0

Set it equal to 1 and solve for w:

If  $w_{T+}$  is less than one (wildtype  $w_{T+} = 1$  fitness) the allele will be lost (near *ρ*=0).

To describe the stability near the point p=0Find  $\frac{\partial p'}{\partial p}$ Substitute in p=0

Set it equal to 1 and solve for *w*:

 $w_{T^+} = 1$  If  $w_{T^+}$  is less than one (wildtype fitness) the allele will be lost (near p=0).

At *p*=1 ...

$$W_{T+} = W_{TT}$$

If  $w_{T+}$  is less than than  $w_{TT}$  the allele will fix in the population (near p=1)

Underdominance has useful spatial properties for testing effector systems.

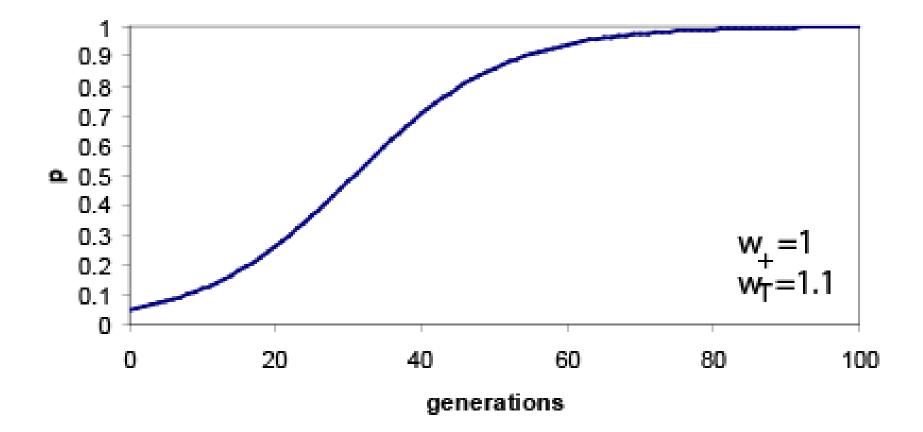
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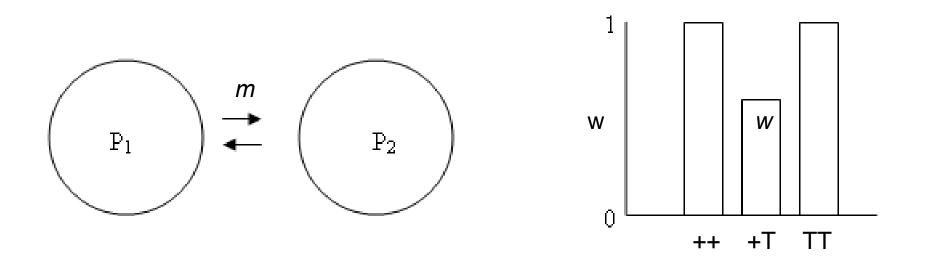
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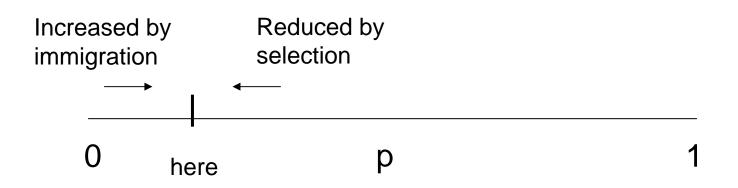
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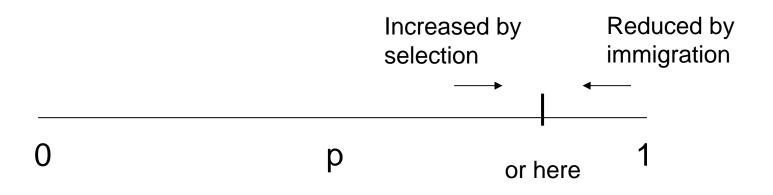
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(TT)$  $(TT) \overrightarrow{\rightarrow} (TT) \overrightarrow{\rightarrow} (TT) \overleftarrow{\rightarrow} (TT) \overleftarrow{\rightarrow} (TT) \overleftarrow{\rightarrow} (TT)$  The simplest model to investigate the spatial properties of underdominance is one of a single locus in two populations exchanging migrants each generation (and equal homozygote fitness).



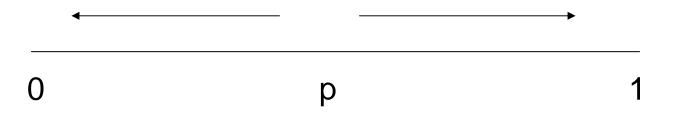
In contrast to the single-population prediction, underdominance can maintain a stable polymorphism among multiple populations connected by migration.



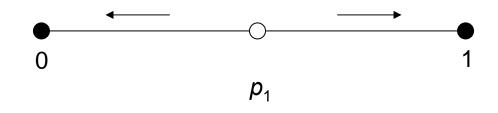
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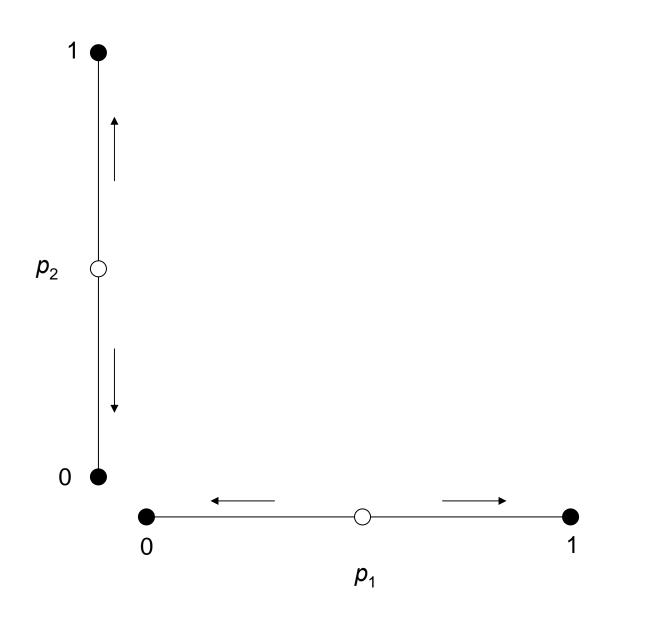


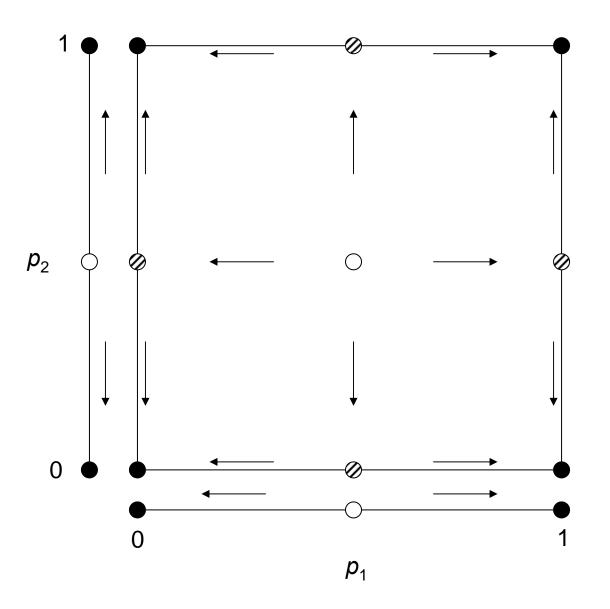
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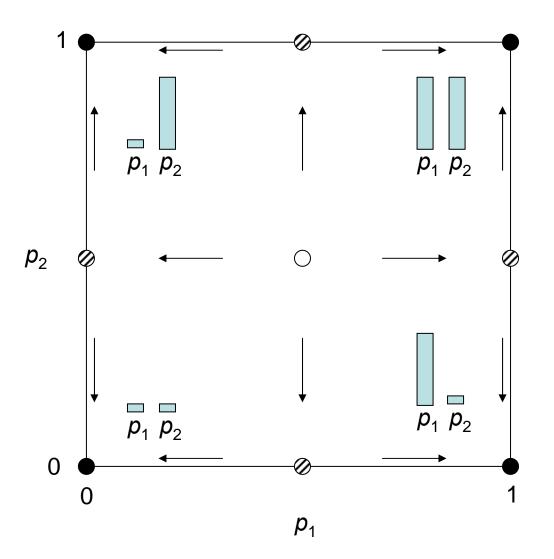


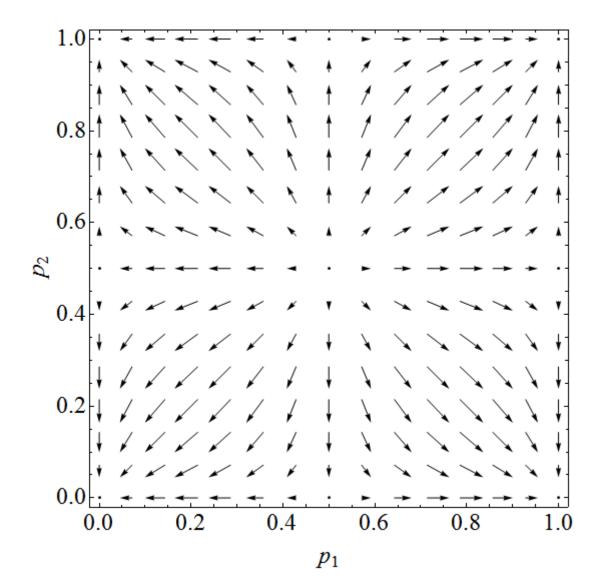
However, if the migration rate is too high, the system behaves like a single population and polymorphism is lost.

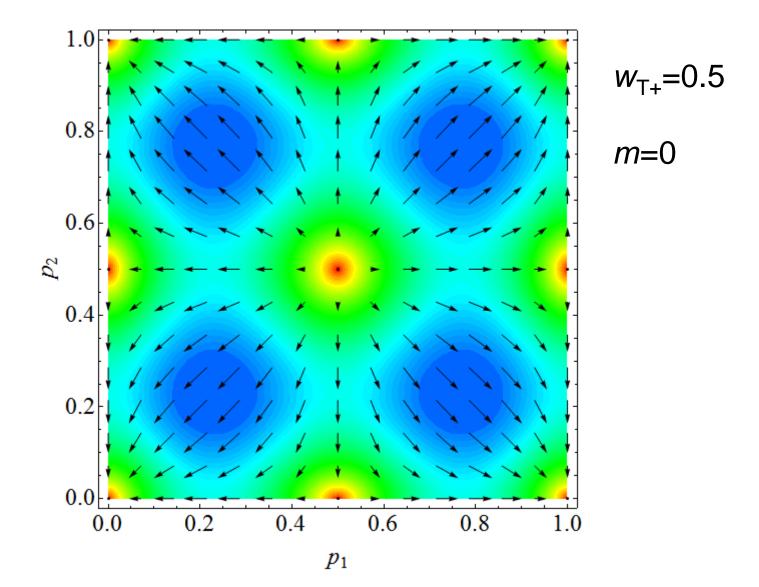


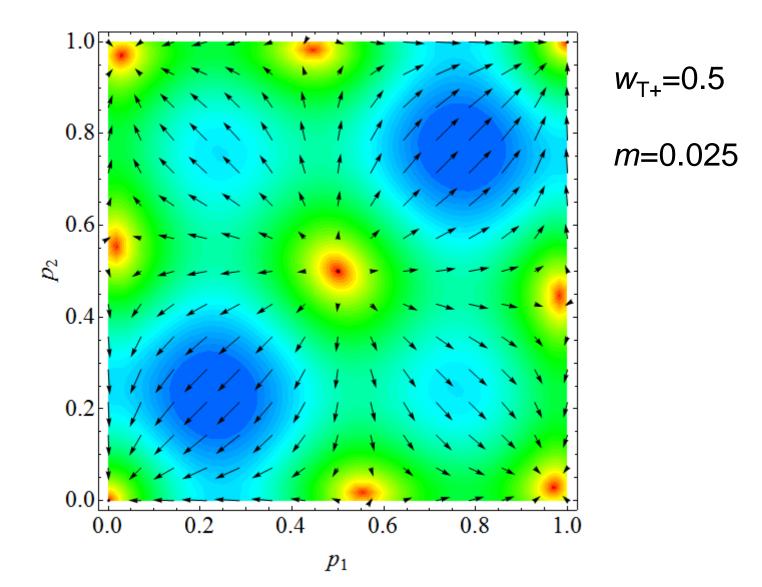


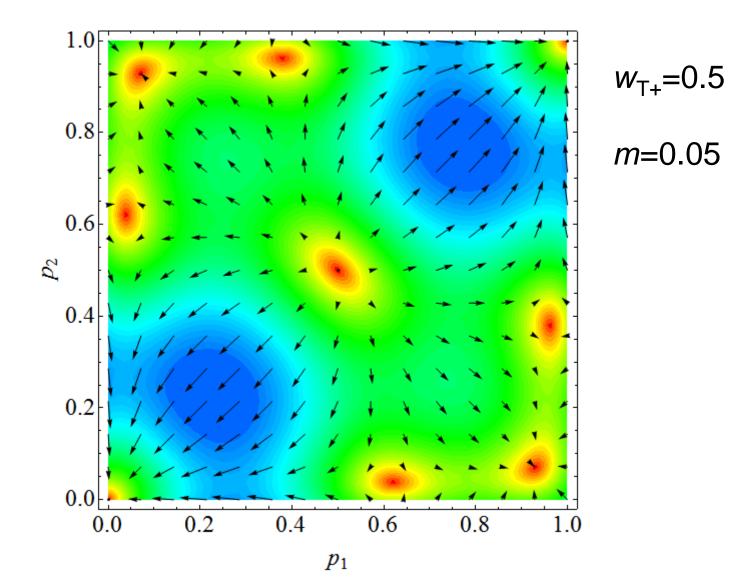


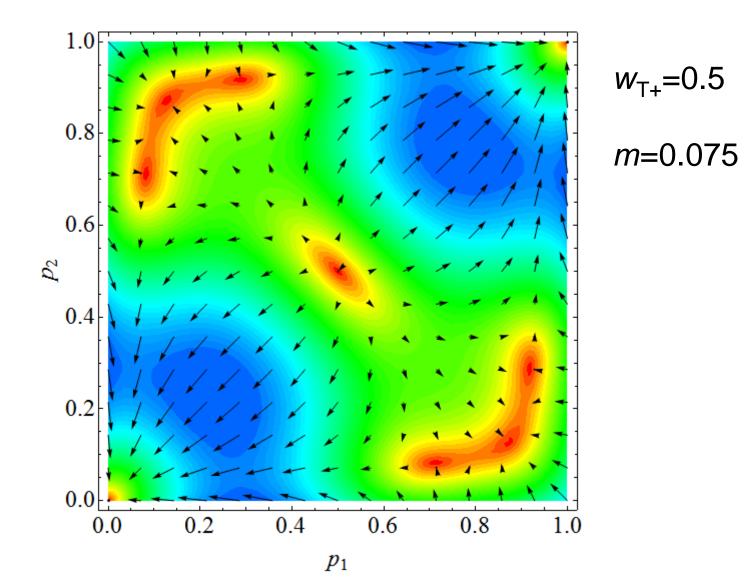


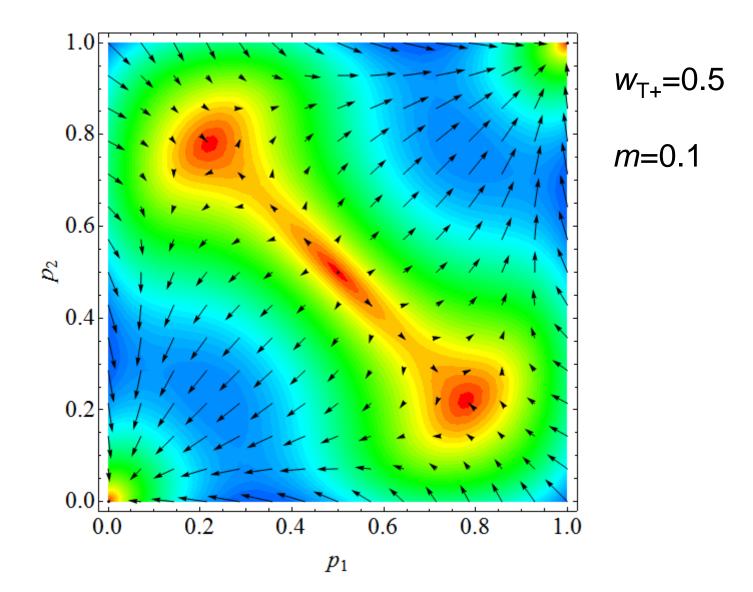


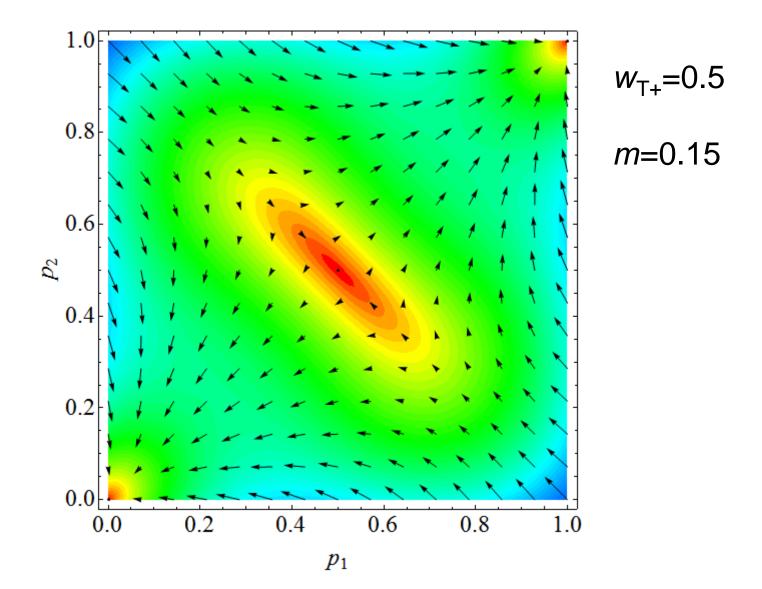


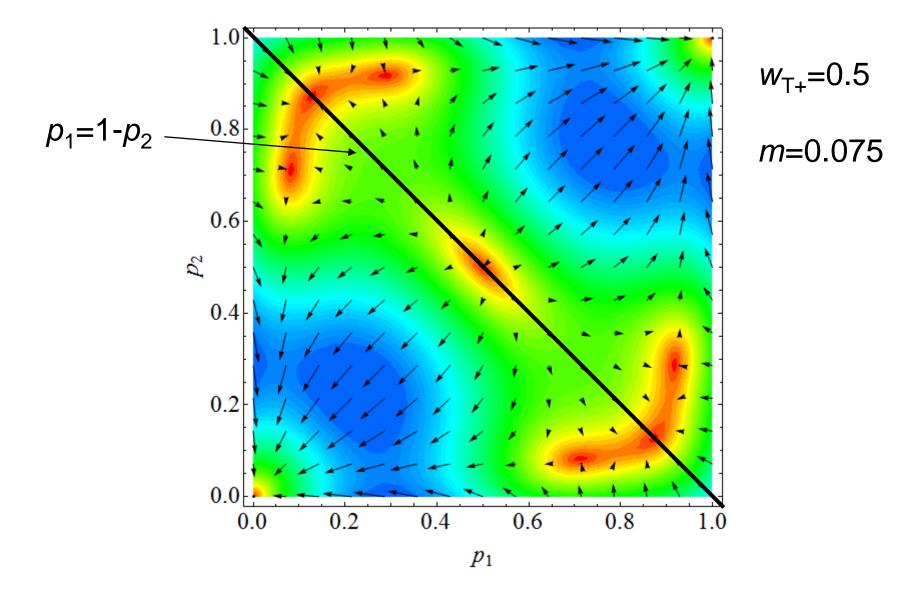












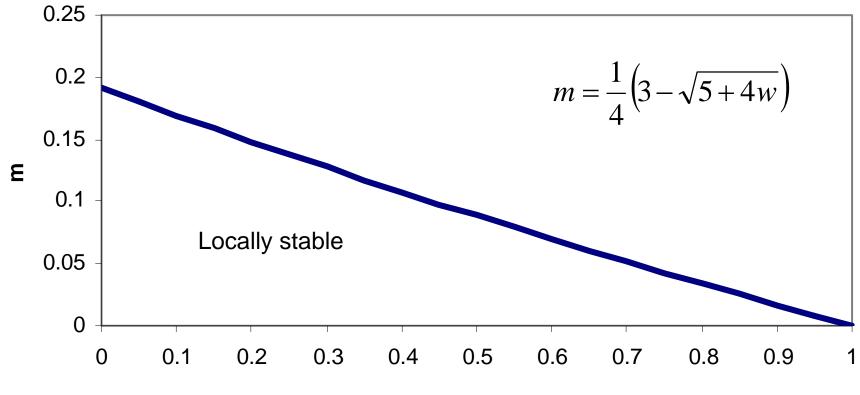
A Jacobian matrix of partial derivatives can be used to create a linear approximation of the system.

$$J = \begin{bmatrix} \frac{\partial p_1'}{\partial p_1} & \frac{\partial p_1'}{\partial p_2} \\ \frac{\partial p_2'}{\partial p_1} & \frac{\partial p_2'}{\partial p_2'} \\ \frac{\partial p_1'}{\partial p_1} & \frac{\partial p_2'}{\partial p_2} \end{bmatrix}$$

Substituting  $p_1=1-p_2$  into the system simplifies it enough to be tractable and the resulting eigenvalues yield the following relationship.

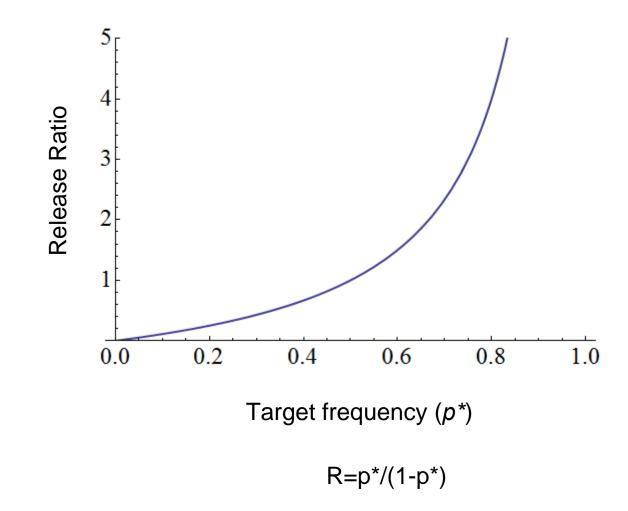
$$m = \frac{1}{4} \left( 3 - \sqrt{5 + 4w_{T+}} \right)$$

The maximum migration rate that allows local underdominant stability.

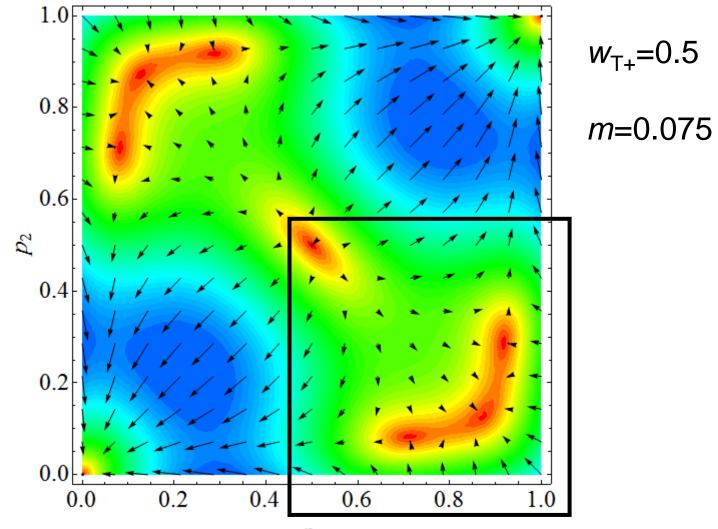


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Strategy to enter the basin.

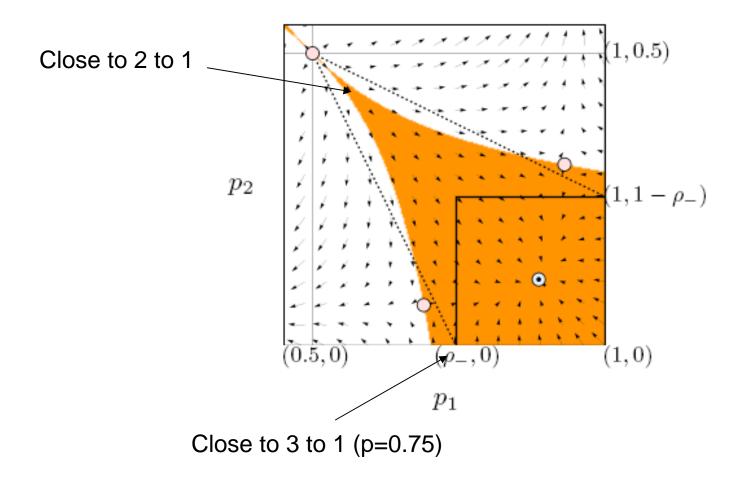


Strategy to enter the basin.

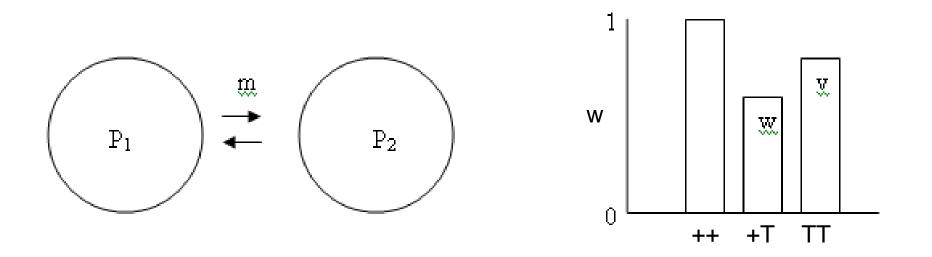


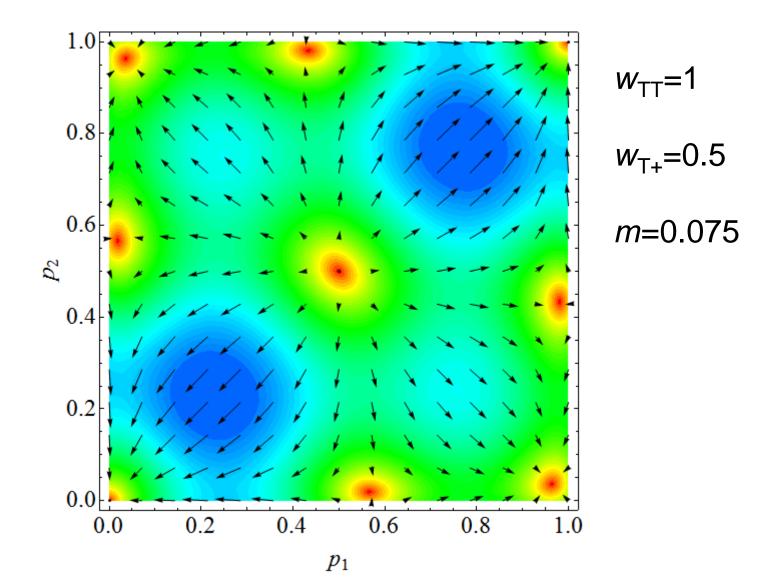
 $p_1$ 

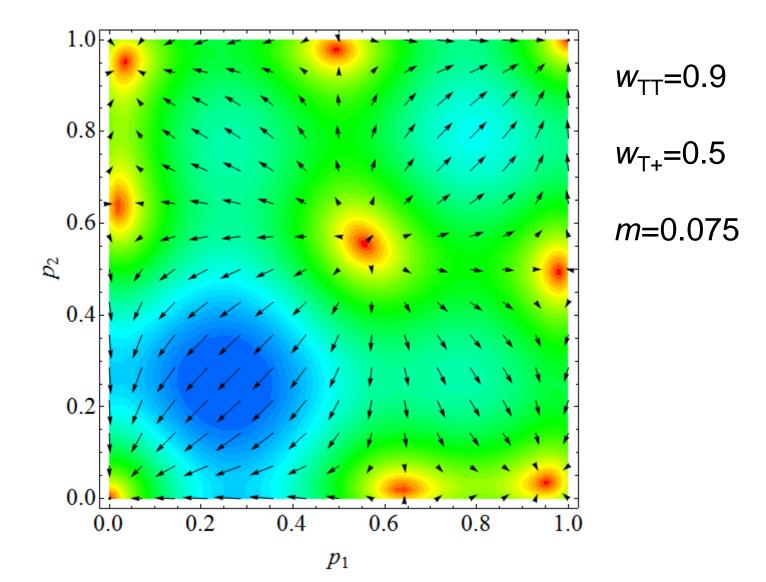
## Strategy to enter the basin.

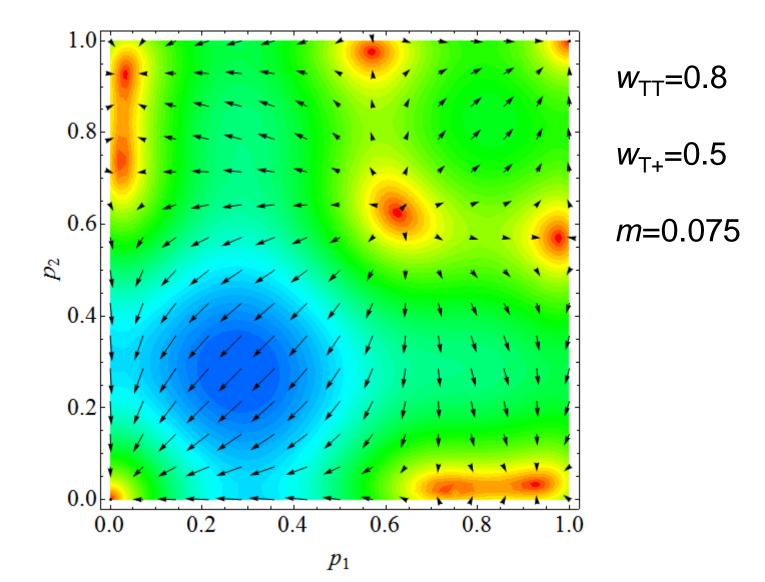


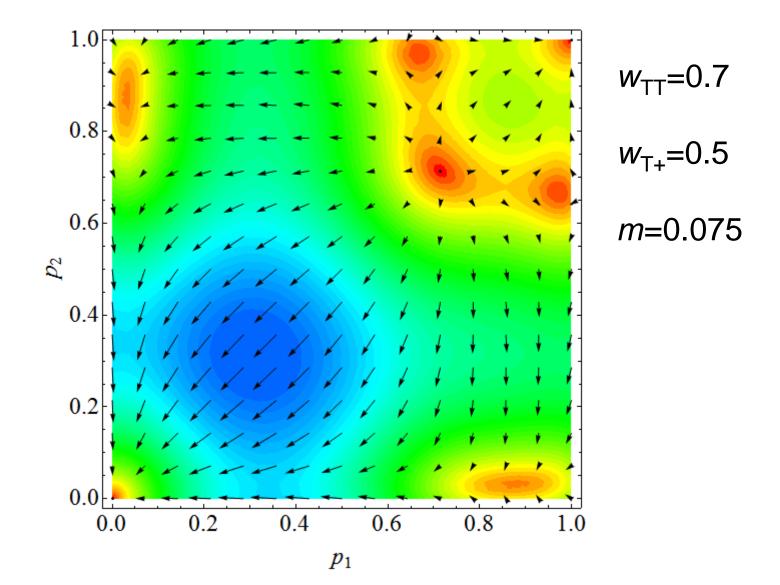
A slightly more complicated model, homozygote fitnesses are not equal.

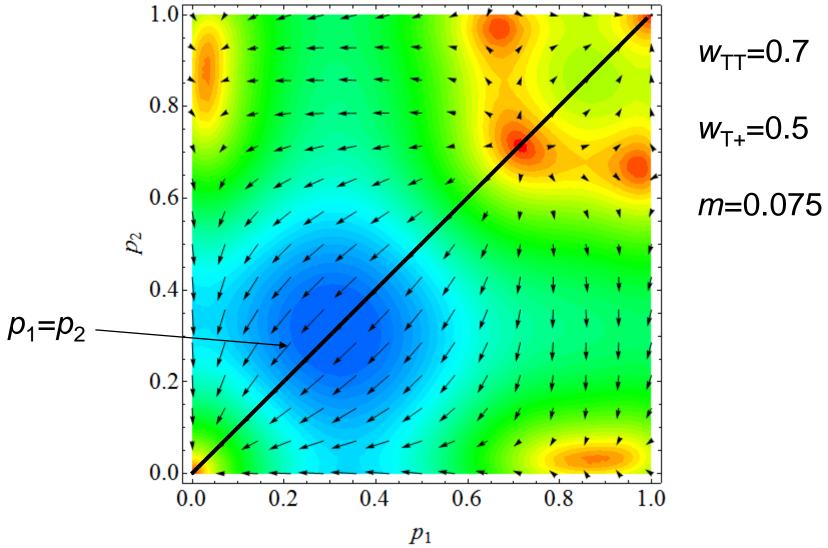








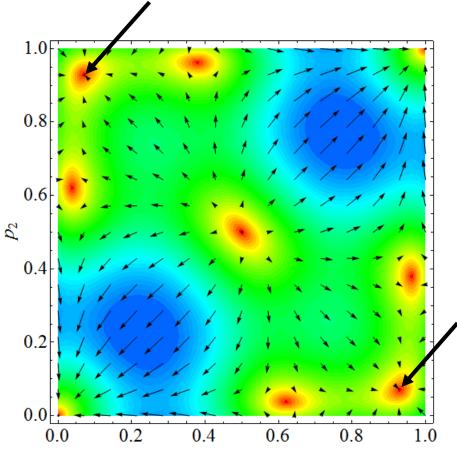




Solving for the unstable equilibrium in this case yields

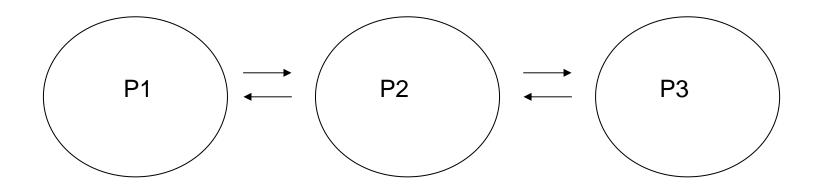
$$\hat{p}_1 = \hat{p}_2 = \frac{w_{T+} - w_{TT}}{2w_{T+} - 1 - w_{TT}}$$

This is independent of the migration rate and equivalent to the single population case given in the introduction. Incidentally, the average fitness at the non-trivial stable equilibrium is independent of the genotype fitnesses and is only a function of the migration rate.  $\overline{w} = 1-2m$ 



 $p_1$ 

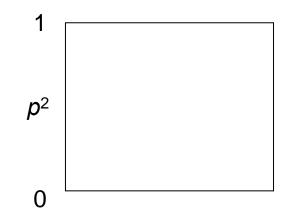
What about more populations?



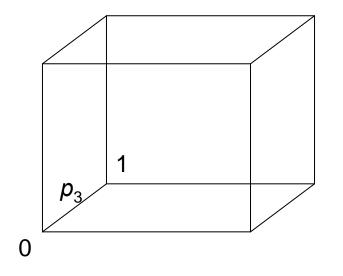
1 population = 1 dimensional system

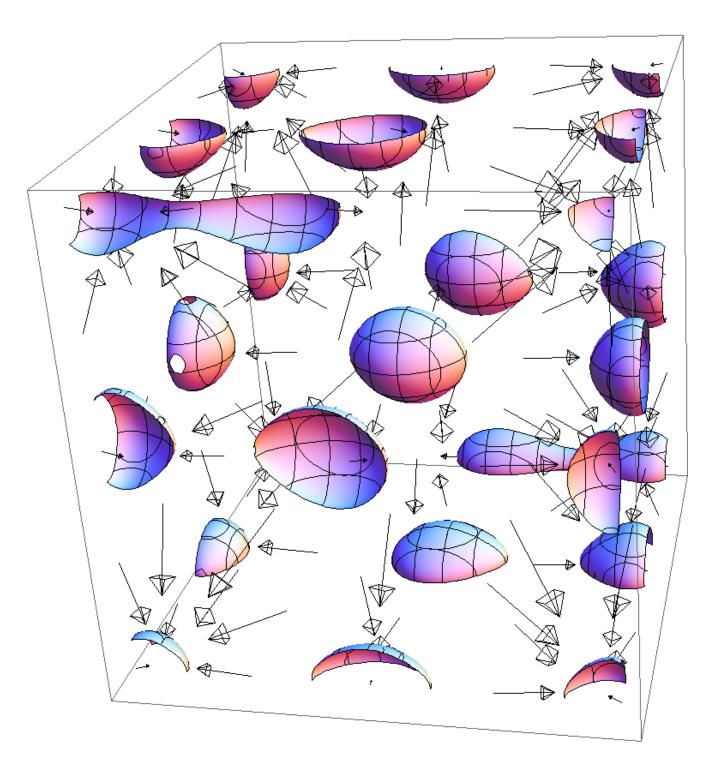
0 *p*<sub>1</sub> 1

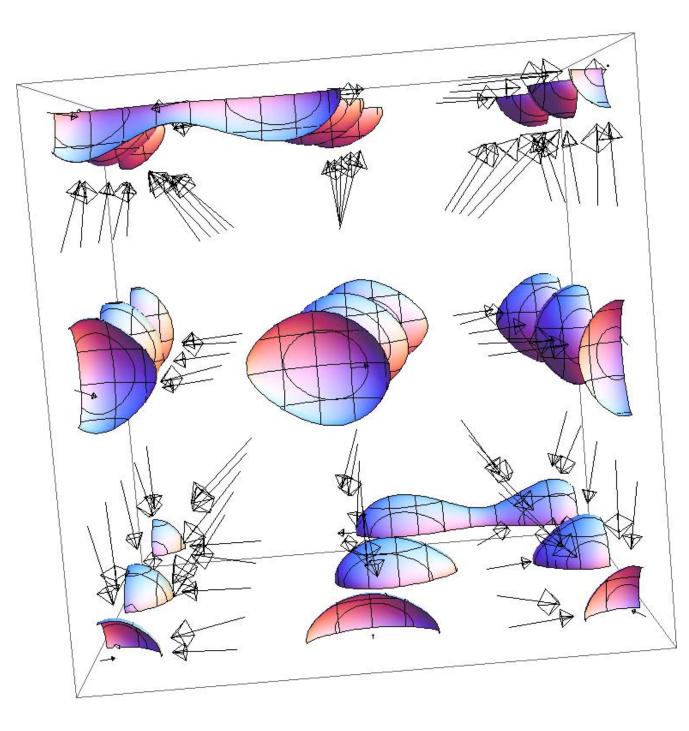
## 2 populations = 2 dimensional system



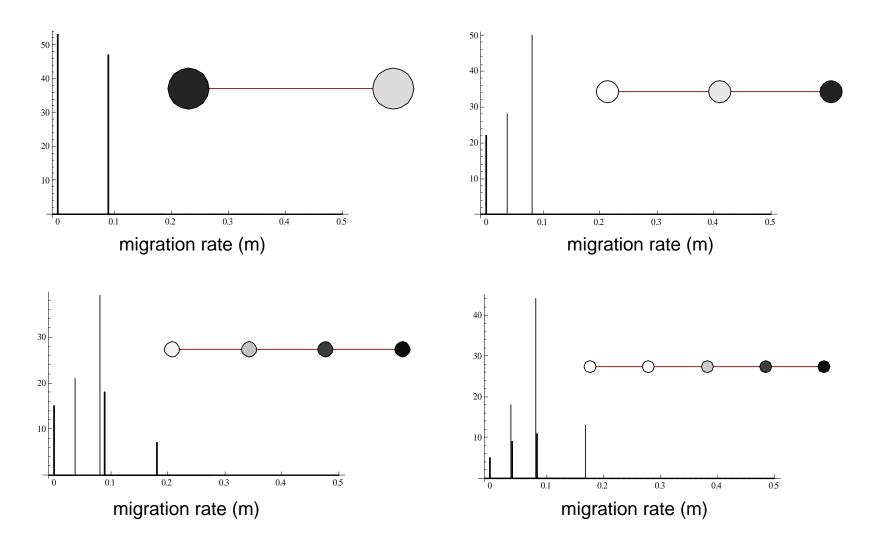
### 3 populations = 3 dimensional system

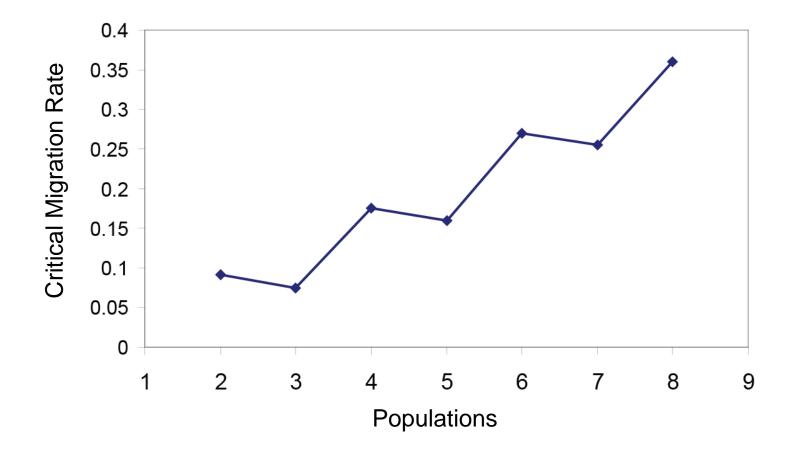




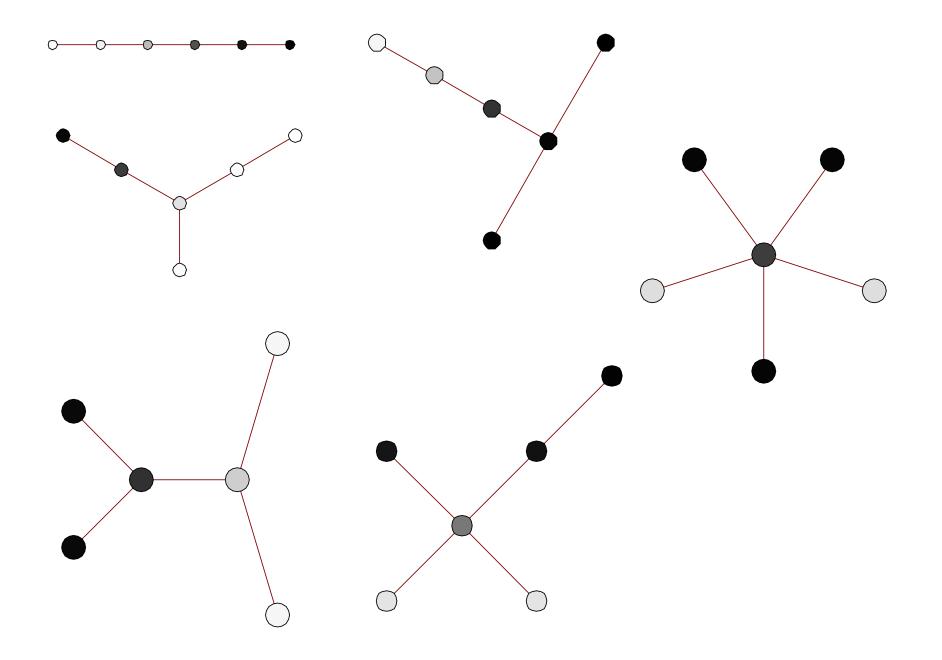


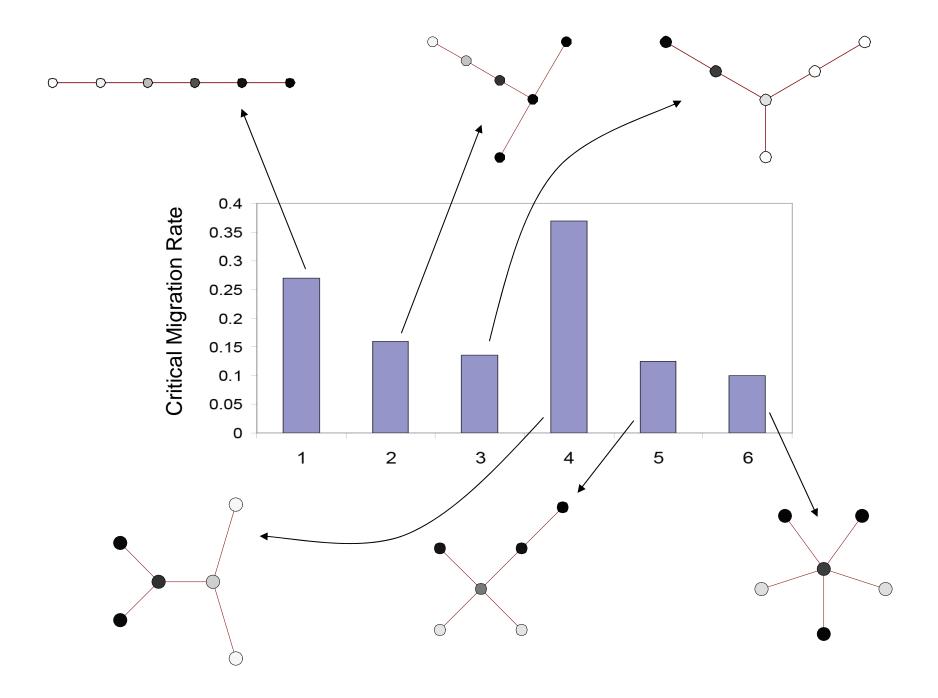
Extending from two populations in one dimension (*w*=0.5).

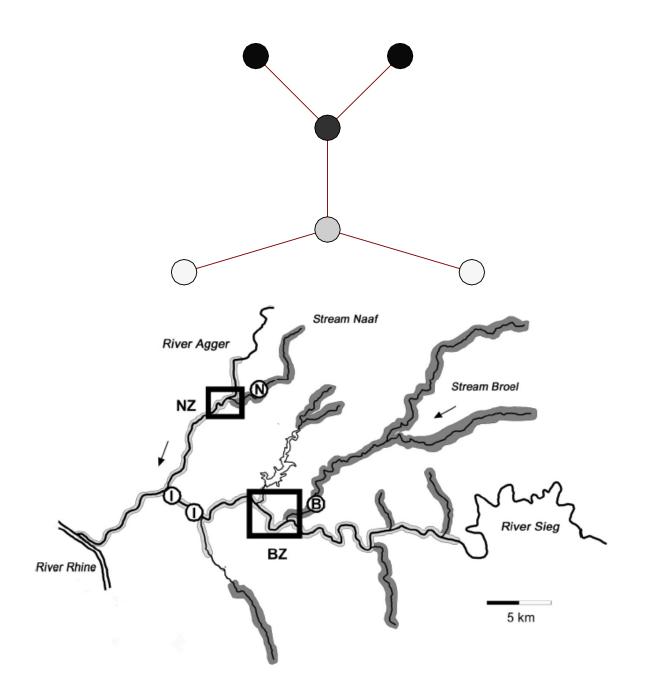




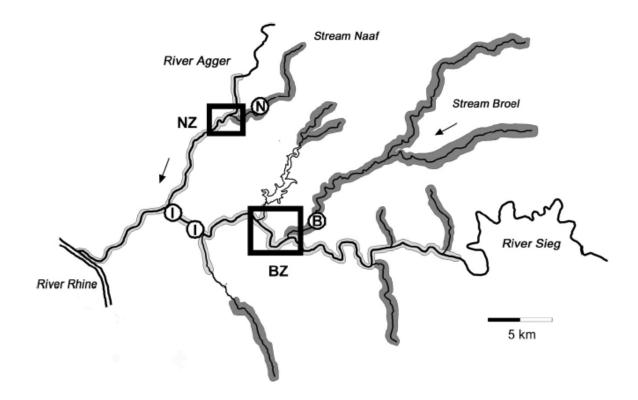
Two buffer populations are better than one or three?













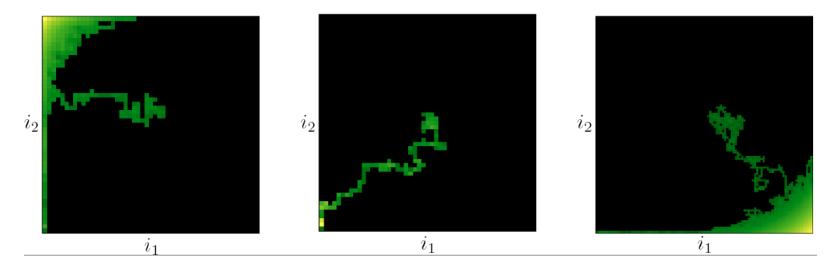
Molecular Ecology (2009) 18, 2615-2627

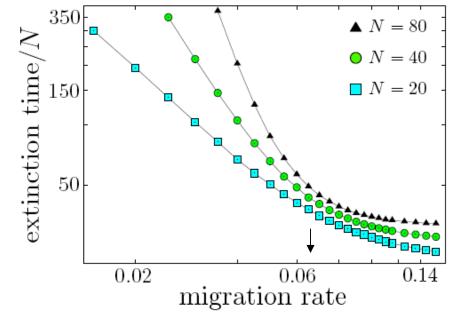
#### Variable patterns of introgression in two sculpin hybrid zones suggest that genomic isolation differs among populations

A. W. NOLTE,\*Z. GOMPERT† and C. A. BUERKLE†

Results suggest widespread underdominance between Cottus populations.

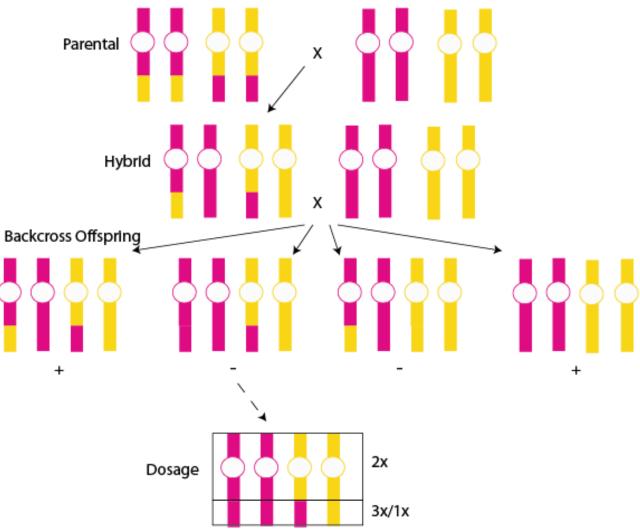
#### **Stochastic/Finite Population Predictions**





How to engineer underdominance?

## Translocations are a Natural Form of Underdominance



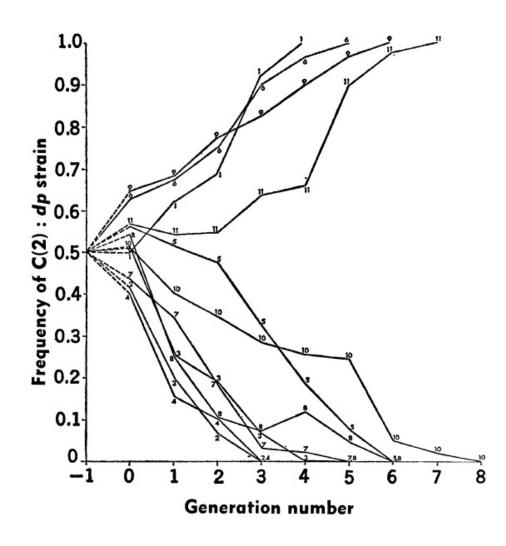
#### Possible Use of Translocations to fix Desirable Genes in Insect Pest Populations

CHROMOSOME translocation heterozygotes (T/+) are usually semisterile, but translocation homozygotes (T/T)if viable are usually fully fertile. If such a viable translocation were produced in an insect pest, T/T insects could be reared in captivity and released into the wild, where matings with wild types (+/+) would produce T/+ progeny.

C. F. CURTIS

Tsetse Research Laboratory, University of Bristol, Langford, near Bristol.

# Early work failed to achieve underdominance with wildtype heterozygotes.



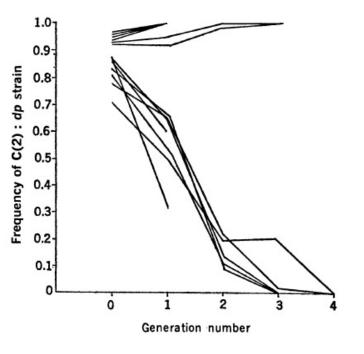
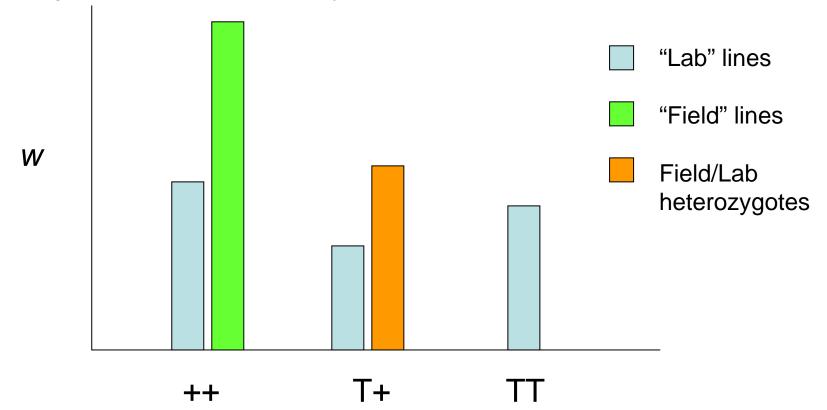


Fig. 6 (left). Change in frequency of C(2): dp strain in competition with C(3): ri strain. The unstable point is about 0.5. Fig. 7 (right). Change in frequency of C(2): dp strain in competition with structurally wild-type strain. The unstable point is about 0.9.

Foster et al. (1972)

Lab lines, especially ones exposed to radiation and made homozygous are much less fit than wild flies (*cf*. Boussy 1988).

(In one dramatic case the lab homozygous fitnesses were so low that there was a heterozygote advantage, despite a translocation!)



#### In recent years other alternatives have been proposed.

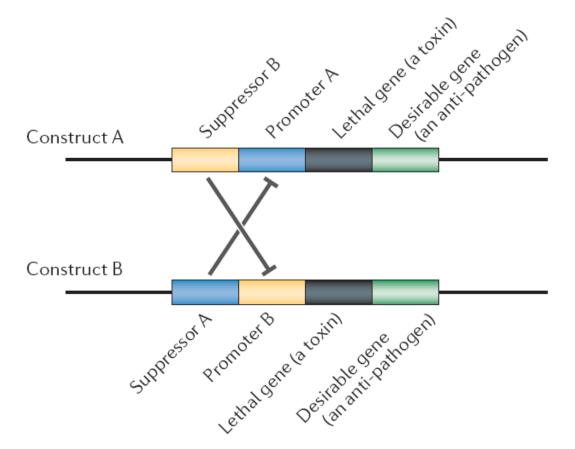
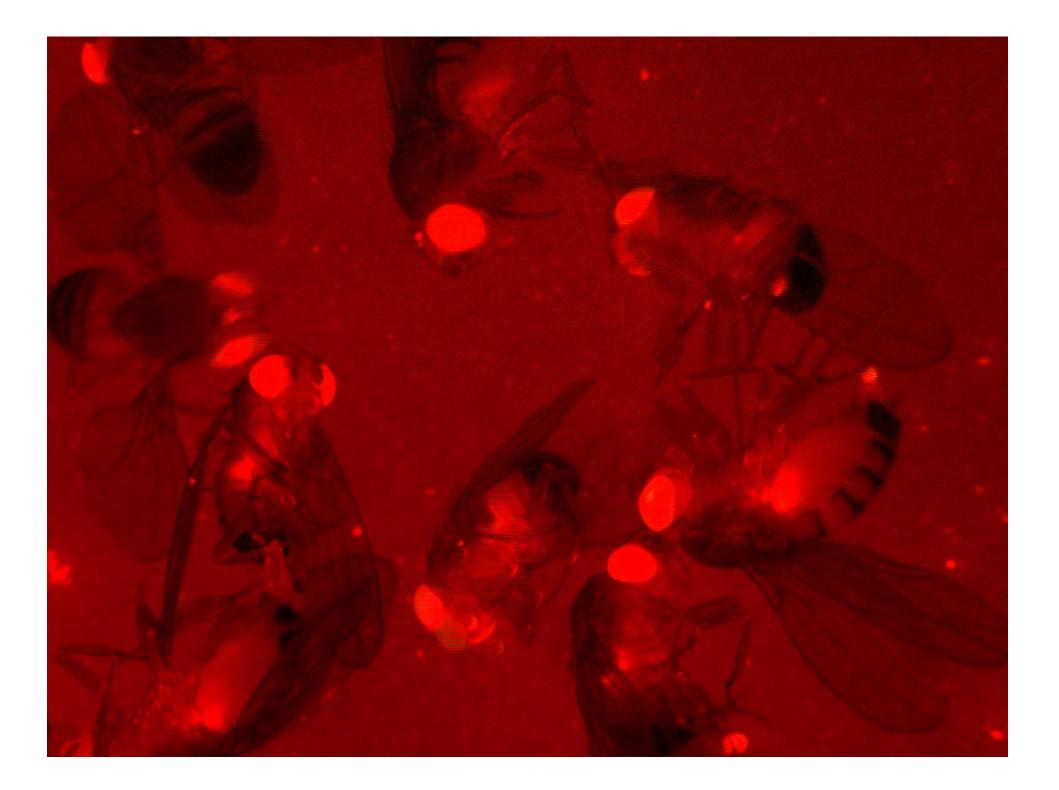


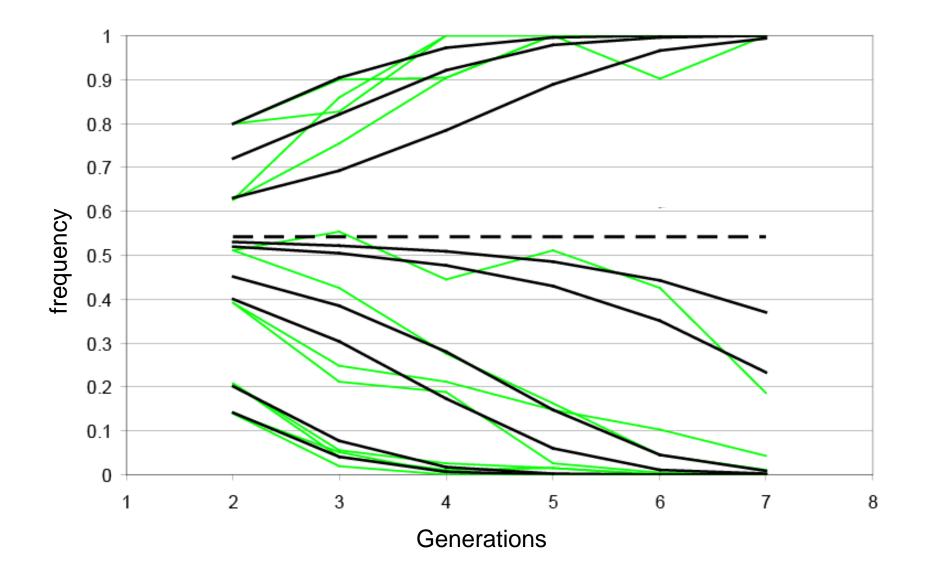
Figure 3 | An example of an engineered underdominant system that is based on mutual suppression of lethal constructs.

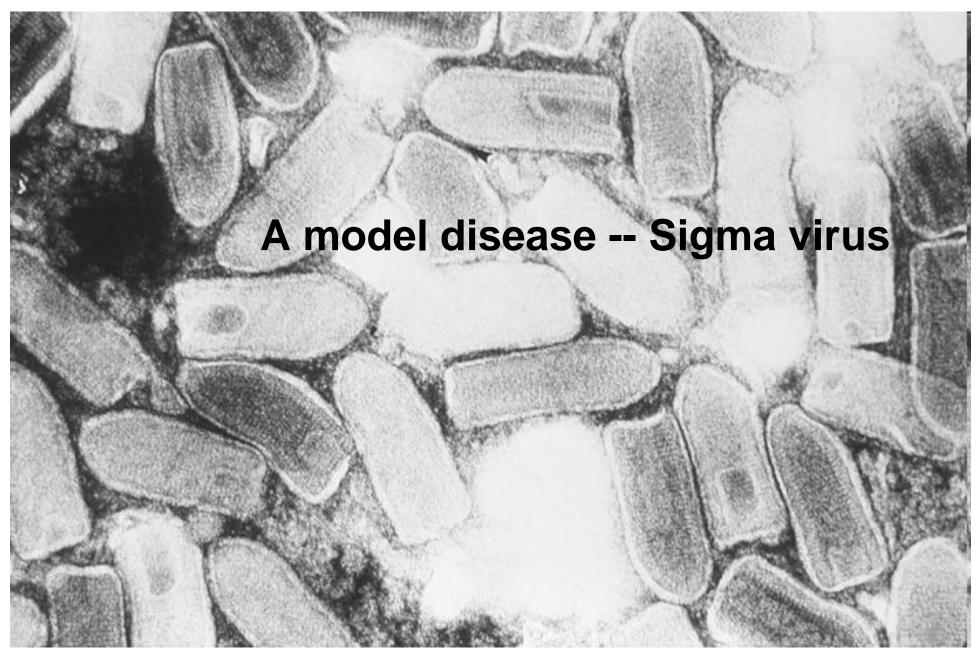
Davis 2001; Sinkins and Gould 2006





### **Theory and Data**





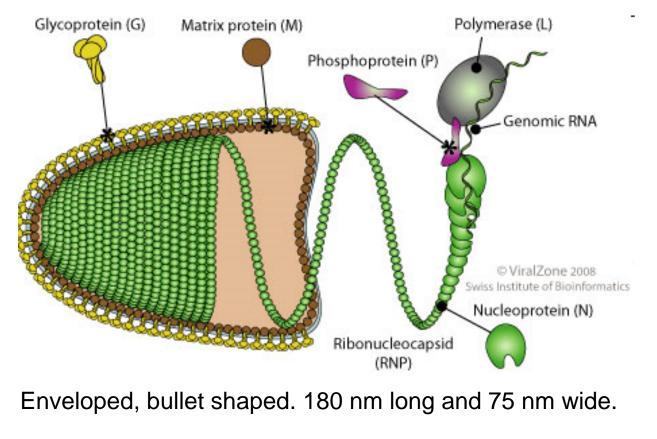
http://micro-writers.egybio.net/blog/?p=97

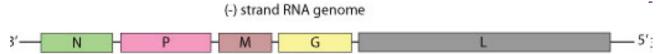
Sigma virus is a Rhabdovirus that infects *D. melanogaster* and is vertically transmitted via the gametes.

Negative sense (3'-5') single strand genome RNA virus.

Rhabdoviruses infect many plants (include crop diseases, some spread by insects), food animals (cattle, trout, salmon, again some spread by insects), and the most well known is rabies.

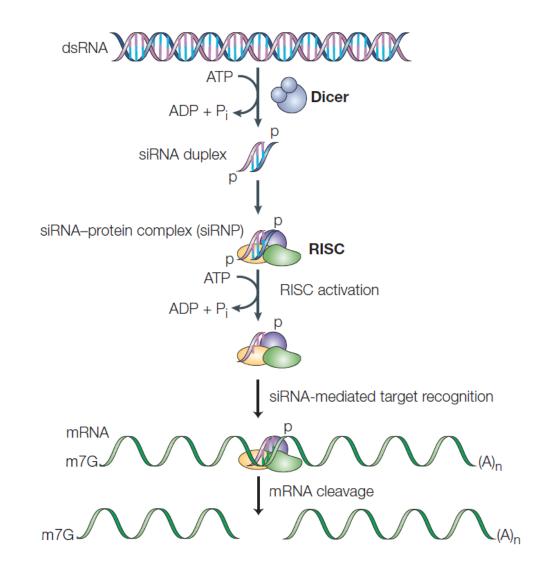
#### **Rhabdovirus Structure**



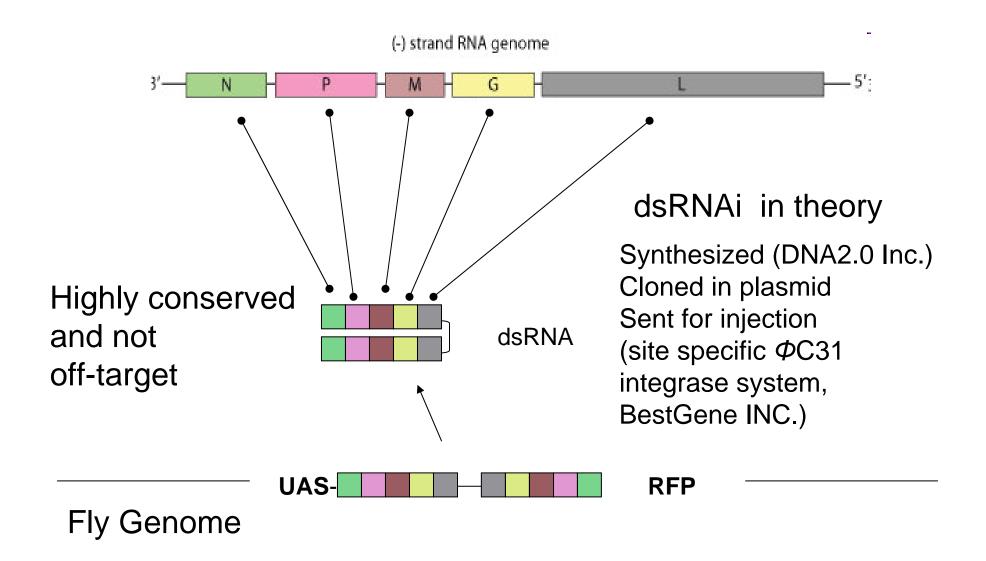


http://www.expasy.org/viralzone/all\_by\_protein/2.html

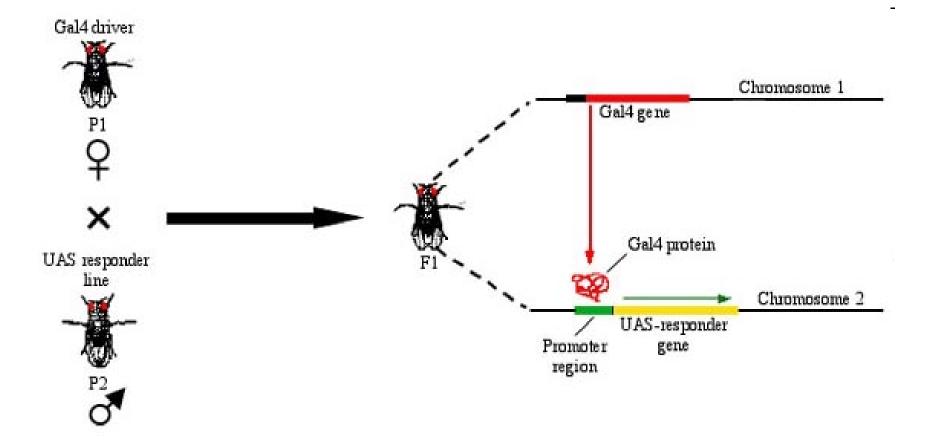
#### Gene expression knockdown by dsRNAi



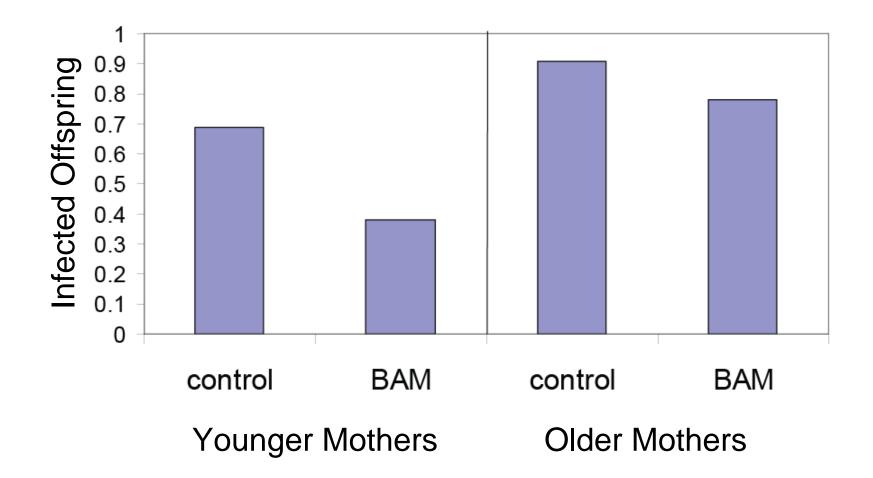
Dykxhoorn et al. 2003



#### GAL4 -> UAS Expression

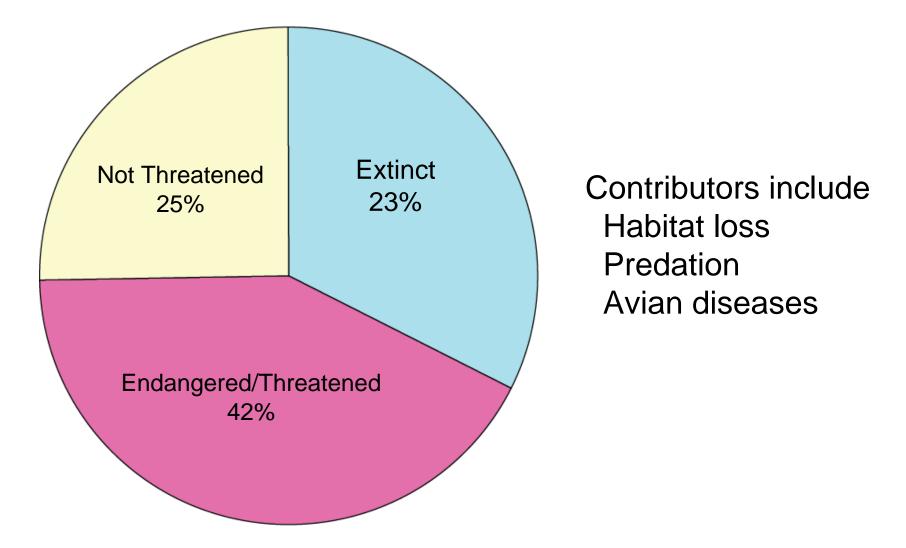


bag-of-marbles expression



A possible conservation application with endemic Hawaiian birds and avian malaria

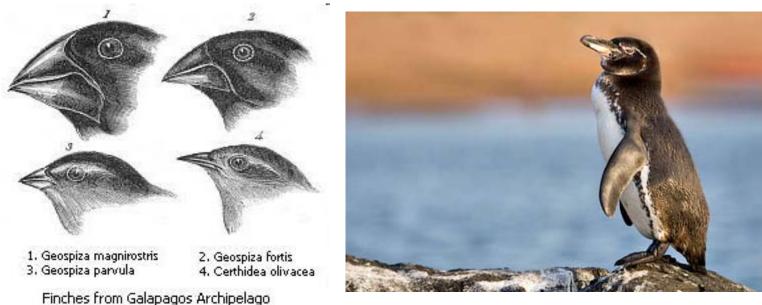
### Status of Endemic Hawaiian Birds





*Culex* mosquitoes were accidentally introduced in the early 19<sup>th</sup> century and are infected with avian malaria.

One bite by an infected Culex is likely to kill a juvenile l'iwi (Atkinson *et al*. 1993) Culex mosquitoes are also now established on the Galapagos islands, and avian malaria has recently been reported there (http://whc.unesco.org/en/news/445, July 1, 2008).



http://www.kiwifoto.com/



Genetic transformation of *Culex* has been possible since 2001.

## Transforming Culex Populations

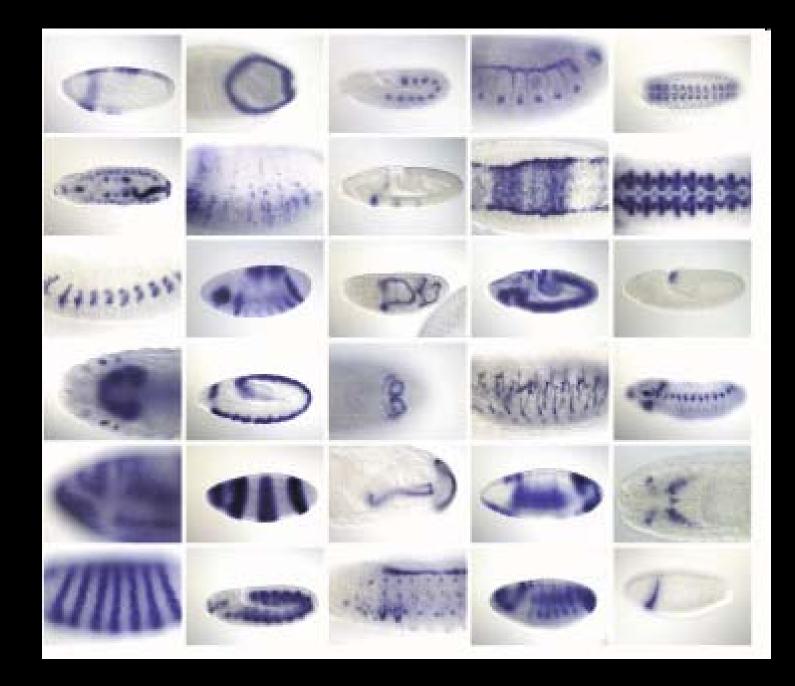
- If an engineered genetic construct gave resistance to avian malaria (*cf.* Jasinskiene *et al.* 2007; Kokoza *et al.* 2010) and could be linked to an underdominant system...
- Releases of transgenic *Culex* that result in a frequency above  $\hat{p}$  could transform an island population to be resistant to avian malaria.
- Parts of the islands could be left untransformed to allow natural resistance in the birds to evolve.
- Underdominance should also prevent the genetic modification from becoming established in the native home range of Culex mosquitoes.

Other interests

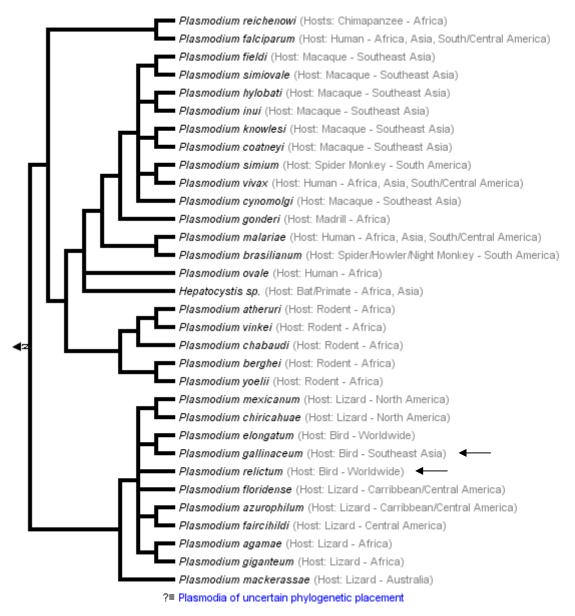
Human evolution/prehistory
Gene-Culture coevolution
Adaptation, selfish genes and selective sweep detection (ABC approaches)
Evolutionary game theory
Hybrid speciation and invasive lineages
Ethics and regulation of GM insects
Genetic sterile insect technique

#### Acknowledgements

Philipp Altrock – Theory Chip Aquadro (Cornell U) – Global Fly Lines Kevin Cook (Indiana U) – Drosophila Genetics Kent Golic (U of Utah) – FLP-FRT translocations Fred Gould (NC State U) – Gene Drive Systems Francis Jiggins (U of Edinburgh) – Sigma Virus Kata Langer – Internship The Max Planck Society – Support Anita Möller – Technician Hagen Müller – Student The Monday Theory Chalk Talk Group (MPI Plön) Guy Reeves – Engineering Diethard Tautz – Support Arne Traulsen – Theory Kata Weiß – Internship



#### Tomancak et al. 2002

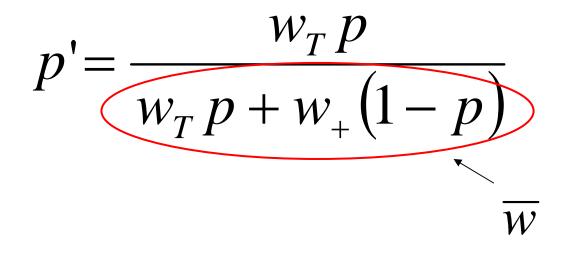


http://www.tolweb.org/Plasmodium

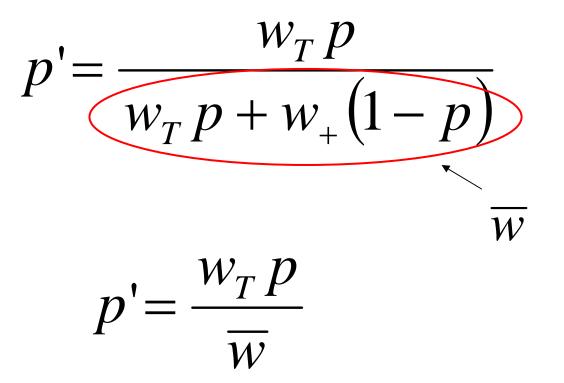
The contribution to the next generation is normalized by the total contribution from both alleles to give an allele frequency in the population.

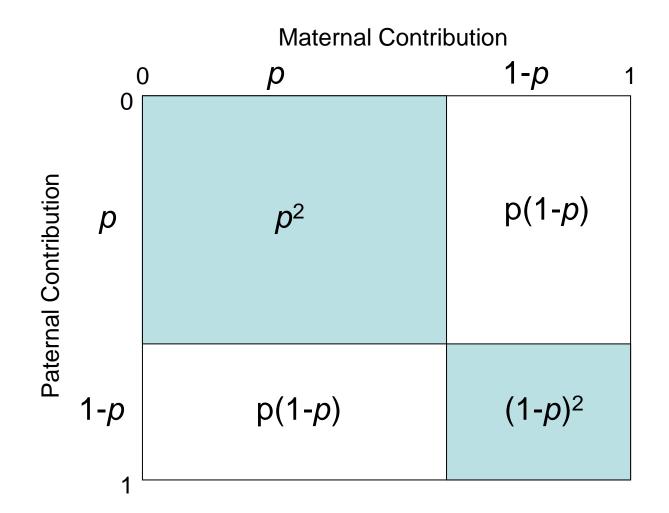
$$p' = \frac{w_T p}{w_T p + w_+ (1 - p)}$$

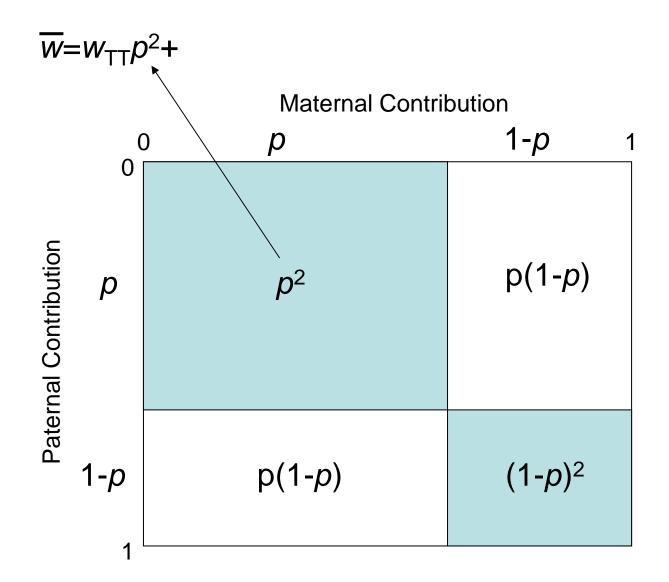
The contribution to the next generation is normalized by the total contribution from both alleles to give an allele frequency in the population.

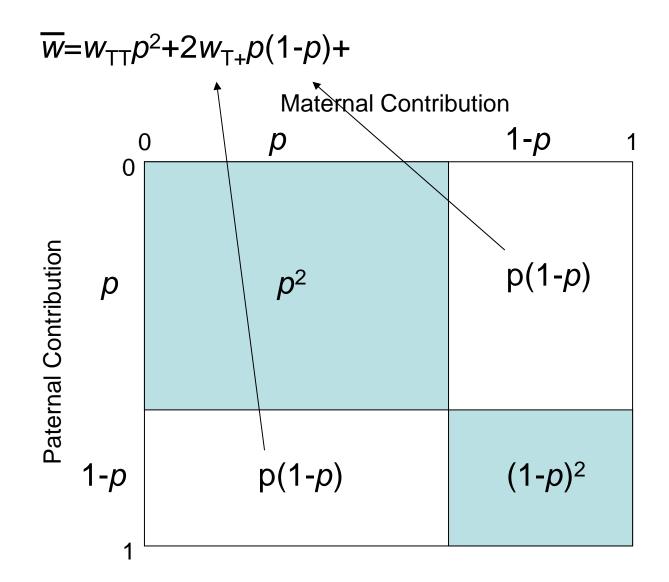


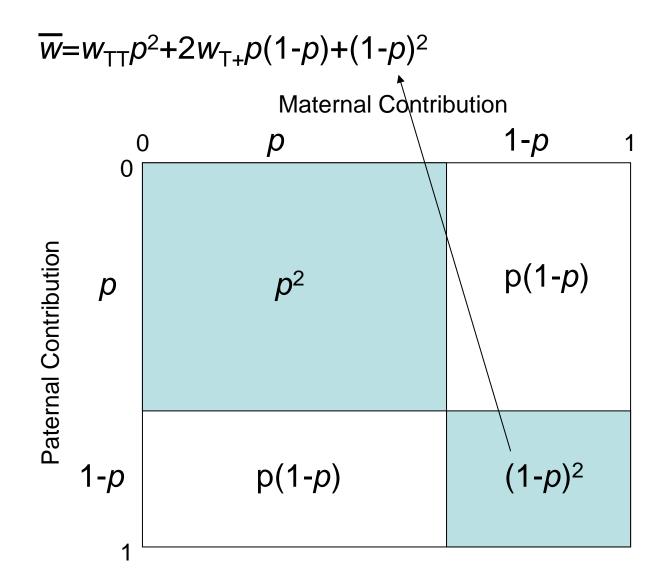
The contribution to the next generation is normalized by the total contribution from both alleles to give an allele frequency in the population.











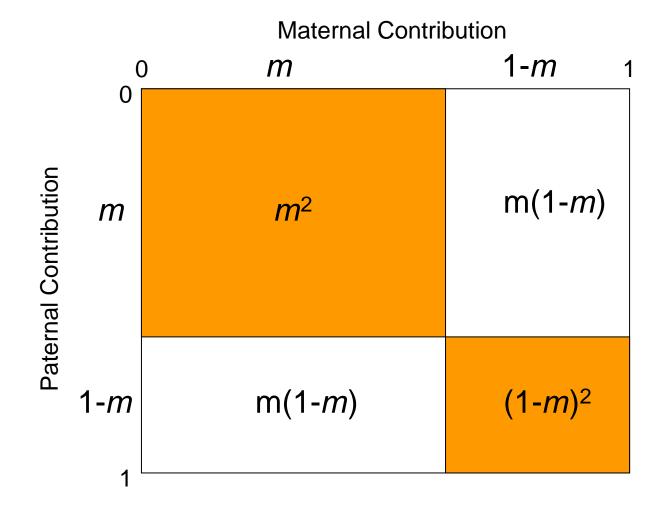
$$\overline{w} = w_{TT} p^{2} + 2w_{T+} p(1-p) + (1-p)^{2}$$

$$p' = w_{TT} p^{2} + 2w_{T+} p(1-p) + (1-p)^{2}$$

$$p' = (w_{TT} p^{2} + w_{T+} p(1-p)) / \overline{w}$$

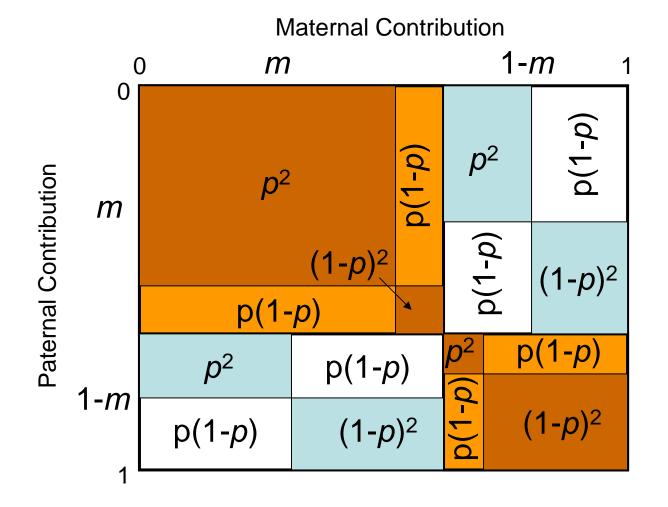
Building the model (on a higher level):

Migrants and non-migrants



Building the model (on a higher level):

Migrants and non-migrants



Building the model (on a higher level):

$$\overline{w}_{1} = (1-m)^{2} (w_{TT} \rho_{1}^{2} + 2w_{T+} \rho_{1} (1-\rho_{1}) + (1-\rho_{1})^{2}) + 2m(1-m)(\dots) + m^{2}(\dots)$$

The expected change in frequency of the underdominant allele each generation can be written in a Wright-Fisher manner.

$$p_{1}' = \frac{\begin{pmatrix} (1-m)^{2} (p_{1}^{2} + wp_{1}(1-p_{1})) + \\ 2m(1-m) (p_{1}p_{2} + \frac{w(p_{1}(1-p_{2}) + p_{2}(1-p_{1}))}{2}) + \\ m^{2} (p_{2}^{2} + wp_{2}(1-p_{2})) \\ \overline{w_{1}} \end{pmatrix}$$

Where

$$\overline{w_1} = (1-m)^2 (p_1^2 + 2wp_1(1-p_1) + (1-p_1)^2) + 2m(1-m)(p_1p_2 + wp_1(1-p_2) + wp_2(1-p_1) + (1-p_1)(1-p_2)) + m^2 (p_2^2 + 2wp_2(1-p_2) + (1-p_2)^2)$$