Special Relativity: Time Distortion

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April 24, 2017

Relativity is an excellent illustration of mathematical and scientific thinking, the threads of connections that occur throughout history, and how works can build upon each other. There is even a historical connection between Einstein's theory of relativity and genetics via Karl Pearson. We should not be reluctant to try to work through and better understand concepts, which often can be broken down into fairly simple steps, which might at first appear to be overly complex and beyond our abilities. With patience and effort it is amazing that how deep under the layers of easily observable reality our understanding can go. The distortion of time, moving at different rates for different individuals, fascinates me. I am writing this to help organize my thoughts in order to try to better understand this.

Relativity, in a sense, goes back to Galileo Galilei (1564–1642), who incidentally was exposed to Pythagorean theory by his father, Vincenzo Galilei, a musician. Galileo's 1632 publication *Dialogo sopra i due massimi sistemi del mondo* was mainly focused on the two theories of the solar system. Did the sun and planets orbit the Earth, or as Copernicus suggested in 1543, did the Earth and planets orbit the sun. Galileo discussed many other things such as William Gilbert's (1544–1603) early work with magnetism and electricity. Galileo also described a situation in which a person who was traveling in a ship smoothly at a constant velocity could conduct physics experiments and not be able to determine the speed the ship was traveling or if it was moving at all—there is no privileged reference frame of absolute motion. This became known as Galilean relativity and was later applied to Newtonian mechanics. Because of this book, which upset intuitive thinking that the sun rose and set around the Earth, Galileo was accused of heresy by the church in 1633, placed under arrest for the rest of his life, and the book was banned until 1835.

Galileo is famous for his discovery of the moons of Jupiter with his telescope in 1610, including the innermost moon Io. The undiscovered planet Neptune also appears in the background of some of his drawings but it was moving very slowly and it is not clear if Galileo thought it was anything but a star.

Galileo mentored a student Vincenzo Viviani who became a professor at the University of Pisa. Vincenzo Viviani had a student, Isaac Barrow, who became a professor at Cambridge. And, Isaac Barrow mentored a student, Isaac Newton (1642–1726) who studied the physics of light among other things.

During the intervening centuries Io was still out there orbiting around Jupiter and Neptune continued on its way, both reflecting light from the sun back to Earth as Earth also continued along its orbit. Galileo's book was still banned but people were constructing new telescopes and conducting new experiments on electricity and magnetism.

One of Isaac Newton's students was Roger Cotes, who mentored a student Roger Smith, who mentored a student Walter Taylor, who mentored a student Stephen Wisson, who mentored a student Thomas Postlethwaite, who mentored a student Thomas Jones, who mentored a student Adam Sedgwick, who mentored a student Charles Darwin (1809–1882) of evolutionary fame. Another of Adam Sedgwick's students was William Hopkins, who mentored a student James C. Maxwell (1831–1879) who studied electromagnetism and another student Francis Galton (1822–1911), a geneticist, who mentored a student Karl Pearson (1857–1936), a geneticist and statistician.

In 1892 Karl Pearson published *The Grammar of Science* which was read by Albert Einstein (1879–1955) and had a profound effect on his thinking. In the book Pearson described the relativity of motion, using geometry to think about physics, and many other things including thought experiments about what happens near the speed of light. Young Einstein became obsessed with imagining what the world would look like if one were able to travel at the speed of light and other scenarios. Pearson speculated that the image of events would be frozen into a constant present.

The theory of light traveling at a certain speed has a long and complicated history. It goes back to Empedocles (ca. 450 BC) in ancient Greece who was influenced by the Pythagoreans. This was modified and improved upon by Ibn al-Haytham in his 1021 publication *Kitāb al-Manāzir*. In 1638 Galileo attempted to measure the speed of light but could only determine that it was either instantaneous or very fast. In 1676 Rømer and Huygens made the first finite estimates of the speed of light, 220,000,000 m/s, by measuring the time of orbit of Galileo's moon Io around Jupiter when the Earth was approaching toward or receding from Jupiter within its own orbit around the sun. This was reported by Newton in 1704 in the book *Opticks*.

Newton also inferred that light of different wavelengths traveled at identical speeds because the eclipse shadows were not colored. This led to a series of experiments of increasing refinements of the speed of light to the modern estimate of 299,792,458 meters per second (in a vacuum).[1]

Newtonian physics could predict the positions of the planets with almost perfect accuracy, within the capabilities of measurements made at the time. However, the orbit of Uranus was constantly off; the planet was not traveling where it should according to Newton. Urbain Le Verrier (1811–1877) hypothesized the existence of another, as yet undiscovered, planet even farther from the sun but close enough to affect the orbit of Uranus with its gravity. Almost immediately Neptune was (re)discovered precisely where Le Verrier mathematically predicted it to be. Following this success, another plant was also off of its Newtonian orbital predictions; Mercury did not transit the sun precisely where it was predicted to. Le Verrier, quite reasonably, predicted the presence of another undiscovered planet even closer to the sun named Vulcan that was having a gravitational influence upon Mercury (Fig. 1). Astronomers searched for the planet Vulcan from 1859 until 1878. There were intermittent reports of possible sightings, some of these were independently replicated by more than one astronomer, but these sighting were challenged as possibly being sunspots mistaken as the outline of a planet back-lighted by the sun and eventually the search was discontinued.

In 1865 James C. Maxwell published *A Dynamical Theory of the Electromagnetic Field* where he provided a set of equations describing the propagation of an electromagnetic field through space.[2] He combined results from previous experiments on electricity and magnetism by people such as Michael Faraday (1791–1867), who discovered that a fluctuating magnetic field produces an electric current, and André-Marie Ampère (1775–1836), who studied how a moving electric current could induce a magnetic field, and synthesized it into a unified model.

Maxwell realized that electromagnetism propagates through space as a self reinforcing wave. Electricity and magnetism are two sides of the

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[1] Increasingly accurate measurements were made by shining light through the "teeth" of a rotating wheel, letting it travel a long distance and reflect back from a mirror, the speed of the wheel can be adjusted until the light does or does not pass back through. Similar experiments were done with rotating mirrors and measuring the angle of reflection given the speed of rotation. This doesn't have to be accepted at face value. The author replicated this kind of experiment in the 1990's. See `https://en.wikipedia.org/wiki/Fizeau%E2%80%93Foucault_apparatus`

[2] Understanding Maxwell's equations takes some work. Here I want to focus on deriving time dilation from the Pythagorean theorem. If you are interested have a look at this site, `http://www.maxwells-equations.com/index.php`.

Figure 1: The planet Vulcan is better known today as the fictional home-world of Spok from the Star Trek series. In the series the Vulcans combine mysticism with logic. However, this was once thought to be a planet in our own solar system. This image is from *Star Trek III: The Search for Spock*, 1984. file link.

same coin and a change in one generates the other. An electromagnetic wave oscillates from a changing electric field that rises and falls, which in turn generates a changing magnetic field that rises and falls, which in turn generates a changing electric field, … *etc.* Once this process is started the wave propagates itself and spreads out through space.

These equations contain the speed of movement of an electromagnetic wave as a constant that does not depend on the relative velocity the source the wave is emitted from or the velocity of a device used to detect it—it is only a process of the trade-off between electric and magnetic fields. If we drop a rock into a pond a wave ripples away from it at a certain speed. The speed of the wave is a property of the surface of the water and does not depend on the size or initial motion of the rock. If the rock is instead a float bobbing to the side at some speed, the waves spread in the direction of movement might be compressed compared to the waves moving in the opposite direction, but (as long as the float is not moving faster than the wave) the wave front will still move outward at the same speed.

Maxwell's equations also allowed the speed of propagation to be calculated from electrical measurements. Maxwell estimated a speed of 310,740,000 meters per second based on the measurement available at the time. This was very close to then current estimates of the speed of light—so close that Maxwell suspected it could not be a coincidence and that light itself was an electromagnetic wave. This suggests that space is like a fluid that electromagnetic waves travel in. This also suggests an experiment that will finally allow the passenger in Galileo's ship to determine how fast the ship is moving. If you drop a rock into water from a moving ship, you can compare the speed of the wave front moving away at different directions from the ship and work out how fast you are traveling and in what direction.

We are moving. The Earth is rotating at 460 meters per second and moving in its orbit around the sun at a rate of 30,000 meters per second. The sun is orbiting through the galaxy at 230,000 m/s and our galaxy is moving through space. What is the sum of all of our motion through the universe?

An experiment was set up so that light traveled in different directions: North and East reflecting back to an observer with a precise timer, or South and West at different times of day, into or away from its orbit. This was carefully tested by the Michelson-Morley experiment of 1887 which bounced the same light source off of mirrors in two different directions at right angles to each other and precisely tested for any difference in speed.[3] The result

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[3] `https://en.wikipedia.org/wiki/Michelson%E2%80%93Morley_experiment`

was that light moved at exactly the same speed in every direction. Either we are back to a pre-copernican view that the Earth is immobile and the entire universe rotates around it (imagine how fast distant stars would have to move through space in order to keep up with rising and setting each night); or we have to accept that the speed of light is invariant and we cannot determine absolute motion. We are back to Galilean relativity; the Earth is a ship and physics measurements are invariant. We cannot determine ultimately how fast and in what direction we are traveling.[4]

However, this is a little strange. Image you place a ruler on the ground next to the edge of a pond. You throw a rock in the water off to the side and measure the rate of movement of the wave across the ruler. It crosses the beginning and end of the ruler at a certain speed. Next you have a friend in a canoe moving slowly in the pond, with the same distance marked off on the side of the canoe, also throw a rock in the water behind the canoe and measure how long it takes for the wave to pass the points marked on the side. It takes longer because the canoe is also moving. From the point of view of the canoe the wave is traveling slower. A wave in the opposite direction would appear to be moving faster; it takes less time to move from one point to the other on the side of the canoe because the canoe is heading into it. Then why didn't the Michelson-Morley experiment detect a difference in the speed of light moving in different directions? How can a wave travel at exactly the same speed, and take the same amount of time to cross two points along a distance on a measuring stick, regardless of how fast or in what direction the measuring stick is moving? One possible answer, as weird as it sounds, is that time is passing slower for your friend on the moving canoe compared to you sitting on the shore. If a stopwatch on the canoe ticks slower, then the wave can cover the greater distance to overtake the canoe in the same "time" as it would take for a wave to cover a shorter distance with a faster ticking clock.[5] The profound insight here is that a wave moves through both space and time and that changes in the rate of time, accompanied with movement through space, might be required to explain why the Michelson-Morley experiment didn't work as expected. However, Newtonian physics did not allow for changes in the rate of time. Yet, Newtonian physics tended to work well and in general was

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[4] Recently is has been determined that we are moving at well over 300,000 m/s relative to the radiation "background" of the universe. However, this is just an average rate compared to distant objects in the universe and not an absolute speed (everything in the universe could be moving together in a direction).

[5] However, this metaphor seems to break down when thinking about the canoe heading into a wave—in that case time would have to tick faster.

very accurate. This led to the proposal of a theory to attempt to integrate a Newtonian description of physics with the constancy of the speed of light.

What is speed? It is the distance moved over a certain time. If the speed of light as a wave moving through space is absolutely constant, regardless of the movement of instruments to emit and detect it, something else has to give. The only options are distance (space) and time. It is profoundly weird and non-intuitive that space and time can bend and are malleable. This goes against our everyday experience. However, so does the notion that the Earth orbits the Sun. In our day to day reality it appears that the sun rises and sets. At its core science is about embracing curiosity, being able to take data and observations seriously, reject our intuition and what may or may not be obvious in our daily experience in favor of logical interpretations of evidence from reality, no matter how strange or non-intuitive it ultimately might be (just as Copernicus did when he realized the Earth orbited the sun, a view that might have also been shared by some ancient Pythagoreans). This is done in order to construct a model of how nature works in order to achieve a better understanding of reality and in order to make new predictions that can be tested. This should be done with a constant constructive criticism, recognition of working assumptions that have been made, and the knowledge that the work is never done; there is always more to discover and refinements to be made. Experience has shown that scientific thinking can be very powerful and predictive (for well tested and developed theories), compared to alternatives such as only relying on intuitive, wishful, or authority thinking.[6]

In our normal day to day lives we form an opinion, hold this as the most important, then search for facts that support it; the individual facts themselves are less important. Science tries to flip this around the other way. Look for facts, which are the most important, then form an opinion that they support, which is less important and disposable if something better comes along.

The Pythagoreans and geometry have been skulking in the background of this article. The Pythagoreans combined mysticism with mathematics to try to explain the universe. They believed that numbers and geometry could be used to unlock secrets of the universe. The Pythagorean theorem is a

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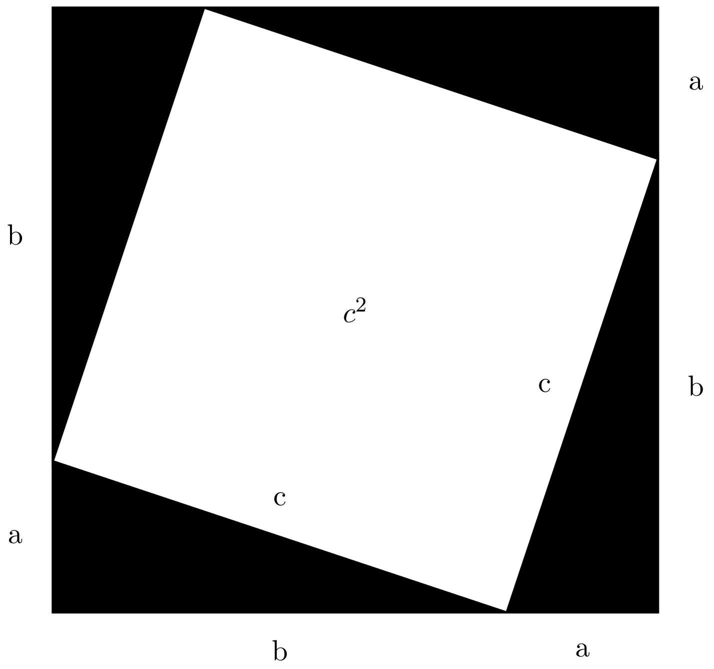
[6] However, none of these exist in a vacuum. Science relies upon intuition and authority (prior knowledge) as a convenience and uses intuition coupled with creativity to advance. We must make choices about what to rely on and what to test. Scientists are not immune to wishful thinking, ego, self interest, *etc.*; however, we hope that we can ultimately overcome fallacies and conceptual dead ends by always holding the thought in some part of our mind that we may be wrong and communicating results and ideas with other people who can then work to challenge and replicate them.

fundamental cornerstone of geometry that establishes a relationship between the sides of a right triangle.

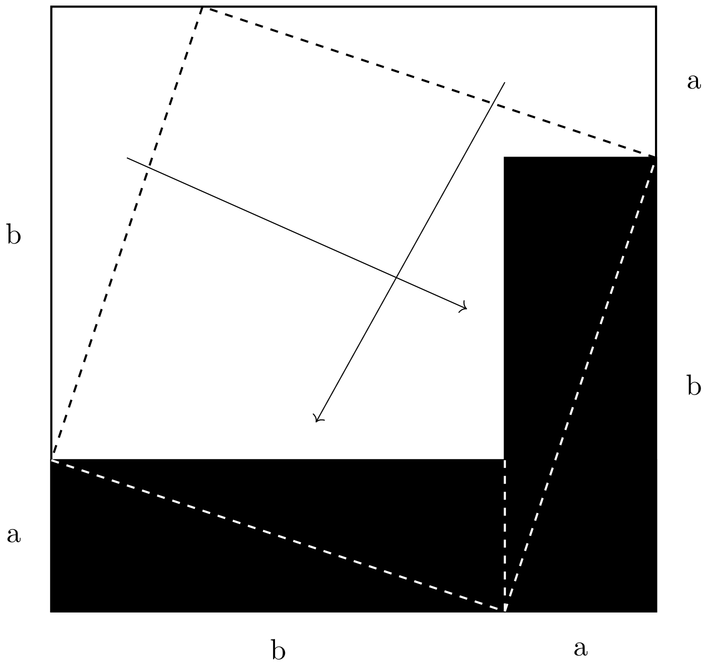
$$a^2 + b^2 = c^2,$$

if c is the longest side of the right triangle, the hypotenuse, and a and b are the other two sides that flank the right angle.

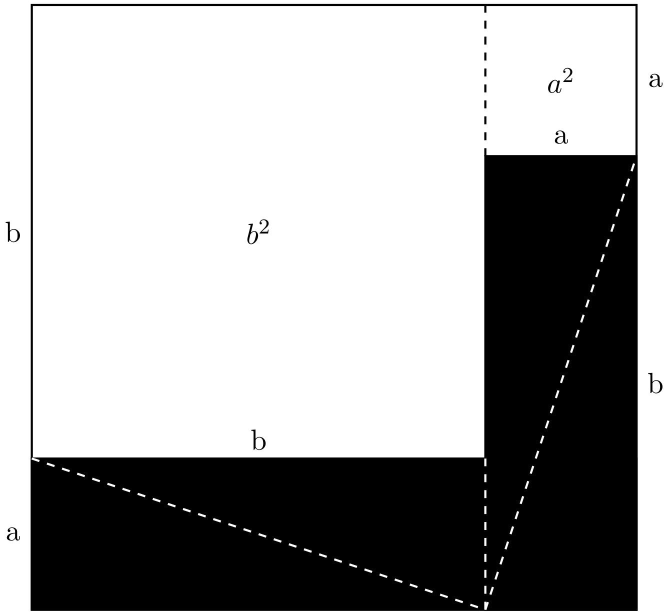
There is an elegant visual proof of this theorem.



The area of the larger square is $(a + b)^2$. The same right triangle has been reproduced around the inside of the larger square. The longest edges make up a smaller square with an area of c^2. We can rearrange the internal triangles to make two more squares.



This rearranges the internal white area into two squares with an area of a^2 and b^2.



The white space inside the larger square has an area of both c^2 and $a^2 + b^2$.

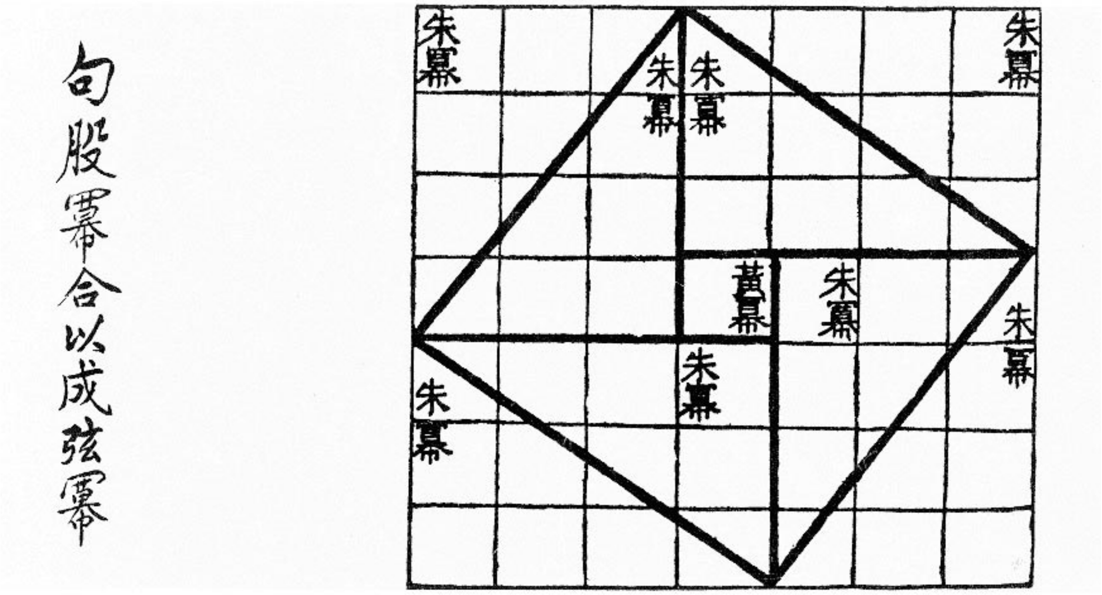


Figure 2: A visual proof of the 3, 4, 5 Pythagorean triple found in the *Zhoubi Suanjing* Chinese text. Translation of the sentence appearing on the left, “The sum of the squares of lengths of altitude and base is the hypotenuse’s length squared.” file link.

Therefore,

$$a^2 + b^2 = c^2.$$

I do need to point out that aspects of the Pythagorean theorem go back many centuries in different cultures and there is a debate regarding if they were discovered independently or knowledge that was shared between ancient cultures. The ancient Babylonians and Egyptians had examples of Pythagorean triples, sets of three numbers, such as 3, 4, 5 or 5, 12, 13, that make up the lengths of right triangles.[7] In India “a rope stretched along the length of the diagonal produces an area which the vertical and horizontal sides make together” appears in the Baudhayana sūtras. China has the “Shang Gao theorem” (Fig. 2).

Okay, now let’s go back to Galileo’s ship, this time moving at a very fast speed in space relative to someone watching the ship through a telescope. Say you set up an experiment to measure the speed of light in the ship.

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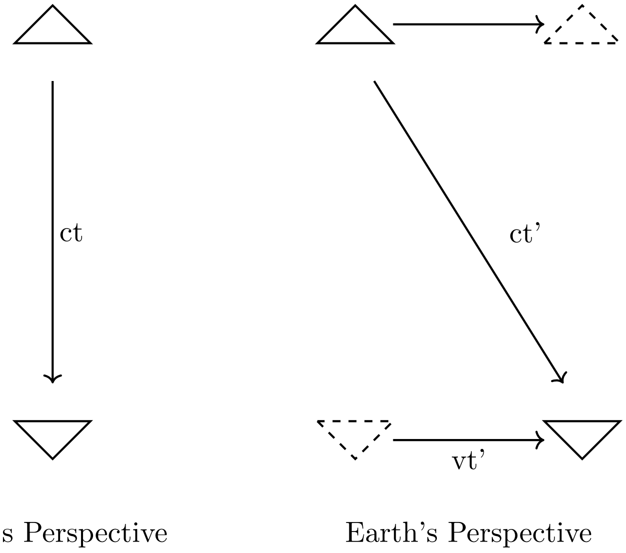
[7] See `https://phys.org/news/2016-04-year-journey-classroom.html` and `http://www-history.mcs.st-and.ac.uk/HistTopics/Babylonian_Pythagoras.html`

You have an emitter and some distance away you have a detector. You measure the time it takes light to move between the emitter and detector (in a vacuum).

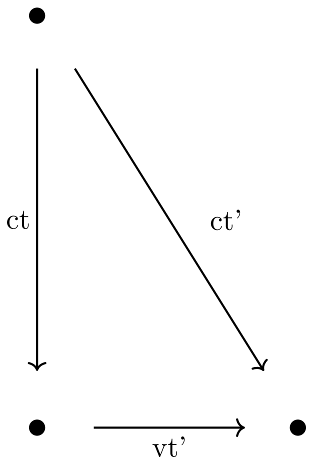
From your perspective the emitter and detector are not moving, because you are moving along with them in the ship. You conduct the experiment and calculate that light is traveling at 299,792,458 meters per second.

Someone on Earth is watching the ship through a very powerful telescope and can see the beam of light as it travels from the emitter to the detector. They run their own calculation and find that it travels at 299,792,458 meters per second; the exact same speed that you found.

However, there is a problem. The light beam traveled over a longer distance from the perspective of the person on Earth. This is because once the light left the emitter, the detector was moving along with the ship, and light had to cover both the distance between the emitter and detector and the distance the detector was moved by the ship.



On the ship the light traveled at the speed of light c over t amount of time. From the point of view of Earth the ship is traveling at a velocity v as a fraction of the speed of light, over a certain amount of time t' that it took the light to travel from the emitter to the detector. Also, from the point of view of Earth the light took ct' to travel. c remains the same so, strangely enough, it is time that has to be different, passing by at different rates t and t', between someone on the ship and someone on Earth.



We have the sides of a right triangle and can use the Pythagorean theorem to calculate the difference in the rate of time between the two observers.

$$(ct)^2 + (vt')^2 = (ct')^2$$

$$c^2t^2 + v^2t'^2 = c^2t'^2$$

$$c^2t^2 = c^2t'^2 - v^2t'^2$$

$$c^2t^2 = t'^2\left(c^2 - v^2\right)$$

$$t^2 = t'^2\frac{c^2 - v^2}{c^2}$$

$$t^2 = t'^2\left(1 - \frac{v^2}{c^2}\right)$$

$$t'^2 = \frac{t^2}{1 - \frac{v^2}{c^2}}$$

$$t' = \frac{t}{\sqrt{1 - \frac{v^2}{c^2}}}$$

And this is the equation for time dilation under the special theory of relativity. Let's isolate part of the equation and set it equal to a new symbol.

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

This γ is known as the Lorentz factor (named after Hendrik Lorentz, 1853–1928). To get the difference in time between a stationary and moving object we can either start with the moving object and multiply by γ

$$t\gamma = t'$$

or start with the stationary object and divide by γ

$$\frac{t'}{\gamma} = t.$$

Pause a moment and think about this. We derived it from ideas and results that pointed towards the invariance of the speed of light, such as Maxwell's electromagnetism and the Michelson-Morley experiment, and the Pythagorean Theorem, which we also derived from a visual proof. We could say that the math is wrong, or that the that the speed of light might actually vary. If you think this might be true you should redesign and conduct the experiments or derivations. I urge you to check my algebra. However, if we accept these results and premises, then fairly straightforward logic leads us to predict a time dilation, which we can quantify and use to compare predictions to observations. Long story short, the predictions work, are accurate, and time does travel at different rates for different observers.[8]

We can plot this as velocity versus the ratio of t'/t and see that if relative velocity is zero then times moves at the same rate, one. However, as we get very close to the speed of light things quickly get strange and much more time has to pass for the observer on Earth relative to the observer on the ship. At a value of approximately 87% the speed of light twice as much time will pass on Earth relative to the ship. At 99% it is seven times as much and at 99.99% it is 70 times as much—70 years pass for every single year—this is serious relativity.

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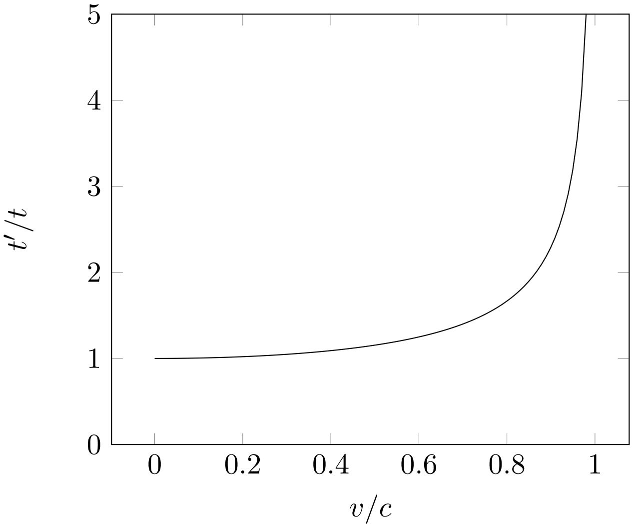
[8] See the 1971 Hafele-Keating experiment for an example, `https://en.wikipedia.org/wiki/Hafele%E2%80%93Keating_experiment`.



The fastest man made object is NASA's Juno spacecraft which was going 73,600 m/s when it arrived at Jupiter in 2016. This is only 0.025% the speed of light, which is a relative time dilation of only 3×10^{-8}. At this rate after one year the difference would only be about one full second less time passing for Juno. This is why relativity seems so strange. In general the effects are tiny in our everyday experience,[9] unless we are very precise in our measurements, and go by undetected.[10]

However, this perspective does lead to a profound redesign of our understanding of the nature of space and time. There is a trade-off between moving through space and moving through time in a combined space-time constant. We trade off moving through time in order to move through space and vice versa. With a little algebra the equations above can be rearranged into

$$(v/c)^2 + (t/t')^2 = 1.$$

The sum of movement through space (as a fraction of light speed) squared

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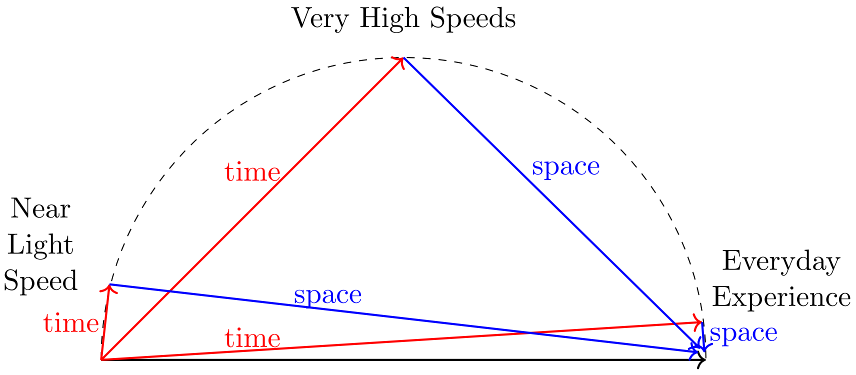
[9] However, relativity does have an influence on our daily lives. If relativity was not taken into account our position, as determined from the reception by our phones of signals by GPS satellites, would accumulate an inaccuracy of approximately 10 km per day! `http://physicscentral.com/explore/writers/will.cfm`

[10] There are of course other fascinating aspects of relativity that I have not touched on here, such a length contraction, the effects of gravity, the relativity of simultaneity, and the infamous $E = mc^2$, which is connected to a Pythagorean relationship: energy^2 = mass^2 + momentum^2 where mass is scaled on a speed of light constant c and can be manipulated by Lorentz factors. I wanted to focus on only one aspect but I encourage you to look into these other topics if it interests you.

and relative movement through time squared is equal to a constant in space-time. In our everyday experience we are moving through space very little and spending most of our space-time budget moving at a right angle through time. However, at high speeds some objects can spend less of space-time moving through time in order to move at a right angle through more space. This invokes Pythagoras one more time. Now the hypotenuse is fixed as a constant and the two sides of the right triangle are space and time, which can be traded off with each other. In the figure below, combined space-time, the center black arrow, is the same for all. However, some can be moving more through time (red arrows) and less through space (blue arrows), while others can be moving more through space and less through time. This trade-off is done by rotating the corner of the right triangle (by speeding up or slowing down our motion through space in the sense that we usually think about it—less familiar is thinking of this as speeding up or slowing down our motion through time). But the hypotenuse of both of these triangles is the same (space-time) distance and phenomena that are embedded within it, such as light, are also the same from any perspective.



Space-Time Constant

This brings to mind what the world might look like if we traveled at the speed of light. Our progress through time would shrink to zero. All experience would be a simultaneous eternal present. Karl Pearson was right!

It is also tempting to wonder what might happen if we were able to travel faster than light . . .

There was a young lady named Bright,
Whose speed was much faster than light,
  She set out one day,
  In a relative way,
And returned home the previous night.

Arthur Buller, 1923

In 1915 Einstein published a general theory of relativity that included the effects of acceleration by gravity. This created an opportunity for experiments possible at the time to directly test new predictions of relativity rather than retroactively designing the rules of relativity to match previous results—this is a much more powerful test of a scientific theory. General relativity predicted that light would be bent by gravity's effects on space-time. The sun is a massive object, 332,946 times the mass of Earth; however, in general it is too bright to be able to see the light from stars nearby, the light of which has to pass close to the sun. An opportunity was coming with a solar eclipse, ironically predicted by Newtonian physics, to block out the light from the sun. A 1919 scientific expedition led by Arthur Eddington (1882–1944) to Príncipe Island off the coast of Africa and simultaneously to Sobral, Brazil to photograph the stars behind a solar eclipse confirmed this. Eddington also went to great lengths to bring an understanding of relativity to the general public through publications, lectures, and radio broadcasts while using humor and allusions.

Oh leave the Wise our measures to collate
One thing at least is certain, LIGHT has WEIGHT,
One thing is certain, and the rest debate —
Light-rays, when near the Sun, DO NOT GO STRAIGHT.

Eddington's parody of *The Rubaiyat of Omar Khayyam*

Let's return to the missing planet Vulcan for a moment exerting its effects on Mercury's orbit. Mercury spends its time very close to the sun. It turns out that this is predicted to have a relativistic effect on the planet's orbit. The predictions of general relativity precisely explained the deflection of Mercury's orbit from Newtonian predictions. The process started by Copernicus and Galileo with additions by Maxwell and others leading to the refinements of Newton's physics by Einstein's relativity along with the work of the Pythagoreans became generally accepted within the scientific community.